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# Singularities and Corners

# Thin inclusions in elastic bodies with possible delaminations

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**Keywords:** thin inclusions, rigid inclusions, elastic body, cracks

Presence of cracks and inclusions in elastic bodies is a source of stress concentrations. We consider the free boundary approach for modeling equilibrium problems of elastic bodies with thin inclusions and possible delaminations. In cases of inclusion delaminations we obtain cracks between the elastic body and the inclusion. Rigidity of the inclusions is one of the parameters responsible for stress concentrations. There are elastic and rigid inclusions as well as inclusions with a zero rigidity (cracks). We consider nonlinear boundary conditions of inequality type at the crack faces to prevent a mutual penetration between crack faces. This approach is much more favorable from the mechanical standpoint as compared to classical linear boundary conditions. Remark that in the case of a rigid inclusion new types of nonlocal boundary conditions appear. This is due to the fact that the inclusion is not fixed, as it is for instance for rigid substrates, but moves rigidly to balance the external forces and the pressure exerted by the surrounding elastic body. Correct mathematical formulations of the problems are proposed. Different problem formulations are considered equivalent to each other. We prove a solution existence for the suitable free boundary problems and analyze other properties of the solution for different locations of thin inclusions with respect to the external boundary. Moreover, we analyze limit cases describing the passages to infinity and zero of the rigidity parameter of the inclusion. In particular, the models of rigid inclusions, semi-rigid inclusions with delaminations and crack models with the non-penetration conditions (inclusions with a zero rigidity) are obtained in the limits.

## References

[1] KHLUDNEV A.M., LEUGERING G., *On elastic bodies with thin rigid inclusions and cracks*, Math. Methods Appl. Sciences, v. 33, N 16, 2010.

[2] KHLUDNEV A.M., *Thin rigid inclusions with delaminations in elastic plates*, Europ. J. Mech. A/Solids, v. 29, N 3, 2010.

[3] KHLUDNEV A.M., LEUGERING G., *Optimal control of cracks in elastic bodies with thin rigid inclusion*, Z. Angew. Math. Mech., v.91, N2, 2011.

[4] ITOU H., KHLUDNEV A.M., RUDOY E.M., TANI A., *Asymptotic behaviour at a tip of a rigid line inclusion in linearized elasticity*, Z. Angew. Math. Mech., v. 92, N 9, 2012.

[5] KHLUDNEV A.M., *Elasticity problems in nonsmooth domains*, Moscow, Fizmatlit, 2010.

[6] KHLUDNEV A.M., *Contact problems for elastic bodies with rigid inclusions*, Quarterly Appl. Math., v. 70, N2, 2012.

[7] KHLUDNEV A.M., NEGRI M., *Crack on the boundary of a thin elastic inclusion inside an elastic body*, Z. Angew. Math. Mech., v. 92, N5, 2012.

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# Graded Mesh Refinement and Error Estimates of Higher Order for DGFE-solutions of Elliptic Boundary Value Problems in Polygons

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**Keywords:** Boundary value problems in polygons, Discontinuous Galerkin Method, Error estimates of higher order

Error estimates for DGFE-solutions are well investigated if one assumes that the exact solution is sufficiently regular. In this talk we consider a Dirichlet and a mixed boundary value problem for a linear elliptic equation in a polygon. Based on regularity results formulated in Sobolev-Slobodetskii spaces and weighted spaces of Kondratiev type we prove error estimates of higher order for DGFE-solutions using a suitable graded mesh refinement near corner points. The main tools are: regularity investigation for the exact solution relying on general results for elliptic boundary value problems, error analysis for the interpolation in Sobolev-Slobodetskii spaces and error estimates for DGFE-solutions on special graded refined meshes combined with estimates in weighted Sobolev spaces. Our main result is that there exist a local grading of the mesh and a piecewise interpolation by polynoms of higher degree such that we will get the same order of approximation as in the smooth case [1].

## References

[1] M. Feistauer, A.–M. Sändig, *Graded Mesh Refinement and Error Estimates of Higher Order for DGFE-solutions of Elliptic Boundary Value Problems in Polygons*, Numerical Methods for PDE, Vol.28, Issue 4, 1124-1151 (2012).

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# Shape sensitivity analysis for a body with a thin rigid inclusion

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**Keywords:** thin rigid inclusions, elastic body, cracks, shape derivative

The equilibrium problem of the elastic body with a thin rigid inclusion is considered. We consider delamination and non-delamination cases. In the case of inclusion delamination we have crack between the rigid inclusion and the elastic body. We suppose that the nonpenetration condition is prescribed on the crack faces. The problems are formulated as variational one. Our goal is to investigate the dependence of the energy functional on the domain variation. For this purpose we introduce a smooth domain perturbation depending on a small parameter. The main result of the work is the deriving of the formula for the shape derivative of the energy functional. Moreover we consider the special cases of the domain perturbations when such derivative can be represent as an invariant integral.

## References

[1] KHLUDNEV A.M., LEUGERING G., *On elastic bodies with thin rigid inclusions and cracks*, Math. Methods Appl. Sciences, v. 33, N 16, 2010.

[2] KHLUDNEV A.M., *Thin rigid inclusions with delaminations in elastic plates*, Europ. J. Mech. A/Solids, v. 29, N 3, 2010.

[3] RUDOY E. M., *Asymptotic behavior of the energy functional for a three-dimensional body with a rigid inclusion and a crack*, J. Appl. Mech. Tech. Phys., v. 52, N 2, 2011.

[4] PLOTNIKOV P.I., RUDOY E. M., *Shape sensitivity analysis of energy integrals for bodies with rigid inclusions and cracks*, Doklady Mathematics, v. 84, N 2, 2011.

[5] ITOU H., KHLUDNEV A.M., RUDOY E.M., TANI A., *Asymptotic behaviour at a tip of a rigid line inclusion in linearized elasticity*, Z. Angew. Math. Mech., v. 92, N 9, 2012.

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# Extracting edge flux/stress intensity functions with the Quasi-Dual Function Method along circular 3-D edges

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**Keywords:** Quasi-dual function method , edge stress intensity functions, penny-shaped crack, 3-D singularities

Asymptotics of solutions to the Laplace equation and the linear elasticity system in the vicinity of a circular singular edge in a three-dimensional domain are provided in an explicit form [1]. These asymptotic solutions are represented by a family of eigenfunctions with their shadows, and the associated edge flux/stress intensity functions, which are functions along the circular edge. We provide explicit formulas for circular cracks for an axisymmetric case as well as a case in which the loading is non-axisymmetric.
Having the explicit solution, we then present a newly developed method, named the quasi-dual function method (QDFM) for the extraction of edge stress intensity functions (ESIFs) along circular crack fronts, in a general three-dimensional domain and boundary conditions. The mathematical machinery was developed in the framework of the Laplace operator in [2]. Here we discuss the elasticity system and apply the QDFM for the extraction of ESIFs from high-order finite element methods.
Three of the advantages of the QDFM are highlighted: a) The possibility to extract the ESIFs away from the singular edge, thus avoiding the need for a refined FE mesh in the vicinity of the singular edge, b) The ESIFs are obtained as a function along the edge and not as pointwise values, c) The method is general in the sense that it is applicable to any circular edge (be it a penny shaped crack, a cylindrical crack or a circular external crack). Several numerical examples are provided that demonstrate the efficiency, robustness and high accuracy of the proposed QDFM.

### References

[1] YOSIBASH Z., SHANNON S., DAUGE M., COSTABEL M., *Circular edge singularities for the Laplace equation and the elasticity system in 3-D domains*, Int. Jour. Fracture, **168**, 31-52, 2011.

[2] SHANNON S., YOSIBASH Z., DAUGE M. AND COSTABEL M., *Extracting generalized edge flux intensity functions with the quasidual function method along circular 3-D edges*, Int. Jour. Fracture, In Press, 2013.

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# Weighted Korn Inequalities for General External Cusps

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**Keywords:** Korn's Inequality, Weighted Sobolev Spaces.

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with $n \geq 2$. It is known [4] that for John domains, the following Korn inequality

$$\|\nabla u\|_{L^p(\Omega)} \leq C\{\|u\|_{L^p(\Omega)} + \|\varepsilon(u)\|_{L^p(\Omega)}\}, \tag{1}$$

holds for displacement fields $u \in W^{1,p}(\Omega)^n$ with $1 < p < \infty$. On the other hand, it does not hold for general Hölder $\alpha$ domains or even for domains containing isolated external cusps [1]. Nevertheless, in those cases, weaker versions of (1) can be proved in weighted spaces involving powers of the distance to the boundary [3] or powers of the distance to the tip of the cusp [5].

For example, for power type cusps of the form $\Omega = \{(x, y) \in \mathbb{R} \times \mathbb{R}^{n-1} : \ \|y\| \leq x^\gamma \ \ \gamma \geq 1\}$ it is shown [5] that

$$\|\nabla u\|_{L^p(\Omega)} \leq C\{\|u\|_{L^p(B)} + \|\varepsilon(u)d^{1-\gamma}\|_{L^p(\Omega)}\}, \tag{2}$$

where $B \subset \Omega$ is a ball, and $d$ stands for the distance to $0$. The authors also show that it is possible to partially "shift" the weight from the right to the left hand side. These results are obtained as a corollary from results for solutions of the divergence operator in weighted spaces.

In this talk we show that (2), as well as the "shifted" versions, hold for the class of general cusps introduced in [2]. That class is defined in terms of some properties of the Whitney decomposition of $\Omega$, allowing sets with poor regularity in $\partial\Omega \setminus \{0\}$. For such an $\Omega$, and following [6], we show that it is possible to extend fields with an $\varepsilon$ preserving operator (an idea that can be traced back to [7]). In our case, the extension works fine only "locally" but allows to reduce the problem to a simple family of domains for which (2) can be shown by means of elementary considerations. Our weights are given in terms of geometric properties of $\Omega$, however for particular cusps of the form $\Omega = \{(x, y) \in \mathbb{R} \times \mathbb{R}^{n-1} : \ \|y\| \leq \phi(x)\}$ can be written using $\phi'^{-1}$.

## References

[1] G. Acosta, R. Durán, F. López García , *Korn inequality and divergence operator: counterexamples and optimality of weighted estimates.* , Proc. of the AMS., 141, 217-232, 2013.

[2] G. Acosta, I. Ojea, *Extension theorems for external cusps with minimal regularity*, Pacific J. of Math., 259, 1, pp 1-39, 2012.

[3] G. Acosta, R. Durán, A. Lombardi , *Weighted Poincaré and Korn inequalities for Hölder $\alpha$ domains.* , Math. Meth. Appl. Sci., 29 (4), 387-400, 2006.

[4] G. Acosta, R. Durán, M. A. Muschietti , *Solutions of the divergence operator on John domains.* , Advances in Mathematics, 26, 2, pp 373-401, 2006.

[5] R. Durán, F. López García, *Solutions of the divergence and Korn inequalities on domains with an external cusp.* Ann. Acad. Sci. Fenn. Math. 35, 421-438, 2010.

[6] R. Durán, M. A. Muschietti, *The Korn inequality for Jones domains.* Electron. J. Diff. Eqns. 2004 (127), 1-10, 2004.

[7] J. A. Nitsche *On Korn's second inequality.* RAIRO-Analyse numerique, tome 15, no 3, p. 237-248, 1981.

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# Improved Boundary Elements for Interface Cracks

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**Keywords:** boundary elements, singularities, Mellin symbol, higher order methods

We present a Galerkin boundary element method to solve the transmission problem with a crack in the interface for the Helmholtz equation. We investigate the regularity for polygons and improve the rate of convergence by various methods: (i) refinement of the mesh towards the crack tips, (ii) augmenting the finite element space by singular functions, (iii) increasing the order of the polynomials for fixed mesh (p-method) [6], (iv) using a geometric mesh refinement together with an appropriate increase of polynomial degree (hp-method) [1, 4]. For other boundary value problems or elasticity problems [3] one can proceed analogously, applying the convergence analysis in [2, 4] to the general method in [5].

## References

[1] BABUŠKA, I., GUO, B.Q., STEPHAN, E.P., *The hp-version of the boundary element with geometric mesh on polygonal domains*, Computer Methods in Applied Mechanics and Engineering 80 (1990) 319-325

[2] M. COSTABEL, E.P. STEPHAN, *Boundary integral equations for mixed boundary value problems in polygonal domains and Galerkin approximation*, Mathematical Models and Methods in Mechanics, Banach Center Publications, Warsaw 15 (1985) 175-251

[3] COSTABEL, M., STEPHAN, E.P., WENDLAND, W.L., *On the boundary integral method for the first fundamental problem in elasticity on polygonal domains*, Recent Trends in Mathematics, Conference in Reinhardsbrunn October 1982, Teubner-Texte zur Mathematik 50 (1982) 56-68

[4] HEUER, N., STEPHAN, E.P., *The h-p version of the boundary element method on polygons*, J. Integral Eqns. Appl. 8 (1996) 173-212

[5] V. PETERSDORFF, T., *Boundary integral equations for the mixed Dirichlet, Neumann, and transmission problems*, Math. Methods Appl. Sci. 11 (1989) 185-213

[6] STEPHAN, E.P., SURI, M., *The h-p version of the boundary element method on polygonal domains with quasiuniform meshes*, Mathematical Modelling Numerical Analysis $M^2$AN RAIRO Analys.Numer. 25 (6) (1991) 783-807

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# Bilaplacian problems with a sign-changing coefficient

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**Keywords:** Bilaplacian, sign-changing coefficient, interior transmission problem

In this talk, we investigate the properties of the following bilaplacian problem:

$$\left| \begin{array}{l} \text{Find } u \in \mathrm{H}^2_0(\Omega) \text{ such that} \\ \Delta(\sigma\Delta u) = f \quad \text{in } \Omega. \end{array} \right. \tag{1}$$

Here, $\Omega$ is a bounded domain of $\mathbb{R}^d$, $d \geq 1$, whereas $\mathrm{H}^2_0(\Omega)$ denotes the closure of $\mathscr{C}^\infty_0(\Omega)$ in $\mathrm{H}^2(\Omega)$. The parameter $\sigma$ is assumed to satisfy $\sigma \in \mathrm{L}^\infty(\Omega)$ with $\sigma^{-1} \in \mathrm{L}^\infty(\Omega)$. Finally, the source term $f$ is chosen in $\mathrm{H}^{-2}(\Omega)$, the topological dual space of $\mathrm{H}^2_0(\Omega)$. Problem (1) appears in the study of Interior Transmission Problems (see the review paper [4]), a family of spectral problems which arise in scattering theory, when one is interested in the reconstruction of an inclusion embedded in a homogeneous medium from multi-static measurements of diffracted fields at a given frequency.

When the sign of the parameter $\sigma$ does not remain constant over the domain $\Omega$, the analysis of the operator associated with problem (1) turns to be not standard because the sesquilinear form $(u, v) \mapsto \int_\Omega \sigma\Delta u\Delta\overline{v}$ is not coercive. In the last two decades, great improvements [6, 7, 3, 1, 2] have been made to understand the properties of the operator $\mathrm{div}(\sigma\nabla\cdot) : \mathrm{H}^1_0(\Omega) \mapsto \mathrm{H}^{-1}(\Omega)$ which occurs when one is interested in the modeling of plasmonic waves or negative metamaterials. In this talk, we prove that $\Delta(\sigma\Delta\cdot) : \mathrm{H}^2_0(\Omega) \mapsto \mathrm{H}^{-2}(\Omega)$ has strikingly different features from those of $\mathrm{div}(\sigma\nabla\cdot) : \mathrm{H}^1_0(\Omega) \mapsto \mathrm{H}^{-1}(\Omega)$. In particular, we demonstrate that $\Delta(\sigma\Delta\cdot) : \mathrm{H}^2_0(\Omega) \mapsto \mathrm{H}^{-2}(\Omega)$ is of Fredholm type as soon as $\sigma$ does not change sign in a neighbourhood of the boundary. We also provide configurations where Fredholmness is lost when $\sigma$ changes sign on $\partial\Omega$. In the process, we study problem (1) with other boundary conditions. For more details concerning these results, we refer the reader to [5, chapter 11].

## References

[1] A.-S. Bonnet-Ben Dhia, L. Chesnel, P. Ciarlet Jr, *$T$-coercivity for scalar interface problems between dielectrics and metamaterials*, Math. Model. Numer. Anal., 46(06):1363–1387, 2012.

[2] A.-S. Bonnet-Ben Dhia, L. Chesnel, X. Claeys, *Radiation condition for a non-smooth interface between a dielectric and a metamaterial*, Math. Models Meth. App. Sci., to appear, 33 pages, 2012.

[3] A.-S. Bonnet-Ben Dhia, P. Ciarlet Jr., C.M. Zwölf, *Time harmonic wave diffraction problems in materials with sign-shifting coefficients*, J. Comput. Appl. Math, 234:1912–1919, 2010. Corrigendum J. Comput. Appl. Math., 234:2616, 2010.

[4] F. Cakoni, H. Haddar, *Transmission eigenvalues in inverse scattering theory*, Inside Out II, G. Uhlman Edition, 2012.

[5] L. Chesnel, *Investigation of some transmission problems with sign-changing coefficients. Application to metamaterials.*, PhD thesis, École Polytechnique, 2012.

[6] M. Costabel, E. Stephan, *A direct boundary integral method for transmission problems*, J. of Math. Anal. and Appl., 106:367–413, 1985.

[7] M. Dauge, B. Texier, *Problèmes de transmission non coercifs dans des polygones*, Technical Report 97-27, Université de Rennes 1, IRMAR, Campus de Beaulieu, 35042 Rennes Cedex, France, 1997. *http://hal.archives-ouvertes.fr/docs/00/56/23/29/PDF/BenjaminT_arxiv.pdf*.

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# Radiation condition and instability phenomenon at a corner interface between a dielectric and a negative material

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**Keywords:** metamaterials, transmission problem, matched asymptotics, rounded corner, Mellin

In the present talk, we consider a 2D second order transmission problem involving a piecewise constant coefficient $\sigma(\mathbf{x})$ in the principal part. This coefficient takes two values $\sigma_\pm$ with opposite signs, which delimits an interface between two media,

$$-\mathrm{div}(\sigma\nabla u) = f \quad \text{in } \Omega, \qquad u = 0 \text{ on } \partial\Omega$$

$$\text{with} \quad \sigma(\mathbf{x}) = \sigma_\pm \text{ for } \mathbf{x} \in \Omega_\pm, \quad \overline{\Omega} = \overline{\Omega}_+ \cup \overline{\Omega}_- .$$

Although this problem may look elliptic at first glance, it is not, due to the sign change of the coefficient. In the case where the interface $\partial\Omega_+ \cap \partial\Omega_-$ contains a corner, which we will assume, the problem is not even Fredholm in a standard Sobolev setting if the contrast between the two media belongs to some critical interval. We will focus on this situation, showing that the problem recovers Fredholmness in a different functional setting, that relies on weighted Sobolev spaces and some outgoing radiation condition at the corner.

We will also discuss recent results of asymptotic analysis that examine the effect of rounding the corner in such a context. We will present an exotic instability phenomenon appearing for our problem as the size for the rounded corner goes to zero.

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## References

[1] A.-S. BONNET-BEN DHIA, L. CHESNEL, AND P. CIARLET JR., *T-coercivity for scalar interface problems between dielectrics and metamaterials*, Math. Model. Numer. Anal., 46(06):1363 1387, 2012.

[2] A-S. BONNET-BEN DHIA, L.CHESNEL AND X.CLAEYS , *Radiation condition for a non-smooth interface between a dielectric and a metamaterial*, Math. Models Meth. App. Sci., in press, 2013.

[3] L.CHESNEL AND X.CLAEYS, S.NAZAROV, *A curious instability phenomenon for a rounded corner in presence of a negative material*, submitted to Asymptotic Analysis, 2013.

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# Profile computations for elliptic problems in domains with small holes

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**Keywords:** Asymptotic analysis, singular perturbation, numerical approximation of elliptic problems, applications in mechanics

Many problems in mechanics or electromagnetics involve bodies with small inclusions or holes. The direct numerical computation of the interesting quantities (displacement, electric or magnetic fields) leadsw to costly simulations since the mesh has to be adapted to the size of the small defects. Many mathematical works have been developed in the past decades to avoid mesh refinemenent, based on asymptotic models involving only the unperturbed domain (and often additional problems in fixed geometries). This is the case of *multiscale asymptotic analysis* trough matched expansions, see [3], or multiscale superposition, see [5], or *homogenization* for periodic networks of holes, see [6].

The problem we will focus on in this talk is quite simple and academic and the basic idea for numerical computations is to replace the solution with its first order asymptotic expansion. To this end, a *profile* needs to be computed in an unbounded domain. We will present several different problems for the Laplace equation and the Lamé equations (for linear elastiticy). Looking for approximate boundary conditions to get a profile problem in a bounded domain may lead to non coercive formulations of Ventcel type. We will address the difficulties encountered and propose solutions as well as numerical computations.

This talk is a survey on joint works with V. Bonnaillie-Noël, D. Brancherie, M. Dambrine, F. Hérau, D. Martin, and S. Tordeux, see [1, 2, 4].

### References


[1] V. Bonnaillie-Noël, M. Dambrine, S. Tordeux, and G. Vial, *Interactions between moderately close inclusions for the Laplace equation*, M3AS, Vol. 10, N°19, pp. 1853-1882, 2009.

[2] V. Bonnaillie-Noël, M. Dambrine, F. Hérau, and G. Vial, *On generalized Ventcel's type boundary conditions for Laplace operator in a bounded domain*, SIAM J. Math. Anal., Vol. 42, N°2, pp. 931-945, 2010.

[3] A. Il'lin, *Matching of asymptotic expansions of solutions of boundary value problems*, Translations of Mathematical Monographs, 1992.

[4] D. Martin, *Mélina : a finite element library*, `http://anum-maths.univ-rennes1.fr/melina/`.

[5] V.G. Maz'ya, S.A. Nazarov, and B.A. Plamenevskij, *Asymptotic theory of elliptic boundary value problems in singularly perturbed domains*, Birkhäuser, Berlin, 2000.

[6] O. A. Oleinik, A. S. Shamaev, G. A. Yosifian, *Mathematical Problems in Elasticity and Homogenization*, Studies in mathematics and its applications. North-Holland, Amsterdam, 1992.


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# On convergent series expansions at a corner in linearized elasticity and Inverse problems

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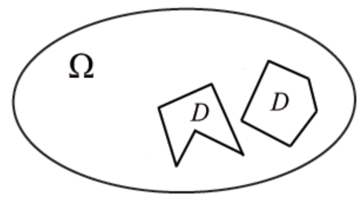
**Keywords:** linearized elasticity, corner, convergent series, reconstruction problem

In this talk, we discuss a convergent series expansion of the displacement field near a corner in two dimensional homogeneous isotropic linearized elasticity. Although the asymptotic expansion of it has been discussed in many literature, its convergence is merely lightly commented. However, when we consider Fracture problems, it is important to analyze the precise behavior of the stress and displacement fields near singular points (cf. [3, 4]). And also it can be applicable to Inverse problems (nondestructive evaluation). Nondestructive testing has been an important technique in gathering information about unknown cracks, cavities or defects in material. From a mathematical point of view, this is described as an inverse problem of a partial differential equation, that is, the problem is to extract information about the location and shape of unknown defects from measured data on the boundary of the body. Then, in the present talk, we consider the following problem.

Let $\Omega$ be a bounded domain of $\mathbb{R}^2$ with Lipschitz boundary and represent a homogeneous isotropic linearized elastic plate. Let $D$ denote cavities embedded in $\Omega$ such that $\overline{D} \subset \Omega$ and $\Omega \setminus \overline{D}$ is connected. Moreover, we require that $D = D_1 \cup D_2 \cup \cdots \cup D_m$, $\overline{D_j} \cap \overline{D_k} = \emptyset$ for $j \neq k$, where each $D_j$ is a simply connected open set and polygon, see for example the figure.




Let $\boldsymbol{n}$ be the unit outward normal vector to $\partial(\Omega \setminus \overline{D})$. Let the displacement vector $\boldsymbol{u} = (u_1, u_2)^{\mathrm{T}} \in \{H^1(\Omega \setminus \overline{D})\}^2$ satisfy the linearized elasticity system in the absence of any body forces

$$\frac{\tilde{E}}{2(1+\tilde{\nu})}\triangle \boldsymbol{u} + \frac{\tilde{E}}{2(1-\tilde{\nu})}\nabla(\nabla \cdot \boldsymbol{u}) = \boldsymbol{0} \quad \text{in} \quad \Omega \setminus \overline{D}$$

and the free traction condition on $\partial D$

$$\frac{\tilde{\nu}\tilde{E}}{1-\tilde{\nu}^2}(\nabla \cdot \boldsymbol{u})\boldsymbol{n} + \frac{\tilde{E}}{2(1+\tilde{\nu})}\{\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^{\mathrm{T}}\}\boldsymbol{n} = \boldsymbol{0}, \quad \tilde{E} = \begin{cases} E \ \text{(plane stress)}, \\ \frac{E}{1-\nu^2}\text{(plane strain)}, \end{cases} \quad \tilde{\nu} = \begin{cases} \nu \ \text{(plane stress)}, \\ \frac{\nu}{1-\nu}\text{(plain strain)}. \end{cases}$$

Here $E$ and $\nu$ are Young's modulus and Poisson's ratio of the elastic medium, respectively. Since both the shear modulus and the bulk modulus are required to be positive, we suppose $E > 0$ and $-1 < \nu < 1/2$.

For this problem, by means of the Mellin transform we derive the series expansion of $\boldsymbol{u}$ near a corner with convergence proof in the form, the details are mentioned in the talk and [2],

$$\boldsymbol{u}(r,\theta) = \sum_{j=1,2,m\in\mathbb{N}} A_{j,m} r^{-s_{j,m}-1} \boldsymbol{\Phi_j}(s_{j,m},\theta) + \sum_{j=1,2,m\in\mathbb{N}} \frac{\partial}{\partial s}\left(r^{-s-1}B_{j,m}(s)\boldsymbol{\Phi_j}(s,\theta)\right)\Big|_{s=\tilde{s}_{j,m}} + \boldsymbol{k}.$$

Moreover, using this formula and the enclosure method introduced by Ikehata [1], we also derive an extraction formula of the convex hull of $D$ from a single set of a surface force and the corresponding displacement field on $\partial\Omega$ [2]. This is a joint work with Masaru Ikehata (Hiroshima University, Japan).

## References

[1] IKEHATA, M., *Enclosing a polygonal cavity in a two-dimensional bounded domain from Cauchy data*, Inverse Problems, **15** 1231-1241, 1999.

[2] IKEHATA, M. and ITOU, H., *Extracting the support function of a cavity in an isotropic elastic body from a single set of boundary data*, Inverse Problems, **25** 105005, 2009.

[3] H. ITOU, A. M. KHLUDNEV, E. M. RUDOY and A. TANI, *Asymptotic behaviour at a tip of a rigid line inclusion in linearized elasticity*, Zeitschrift für Angewandte Mathematik und Mechanik, **92** 716-730, 2012.

[4] HIROMICHI ITOU, VICTOR A. KOVTUNENKO and ATUSI TANI, *The interface crack with Coulomb friction between two bonded dissimilar elastic media*, Applications of Mathematics, **56** 69-97, 2011.

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# Singular Solutions and Large Solutions to some Nonlinear Elliptic Equations in Polygonal Domains

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**Keywords:** Nonlinear elliptic problem, Corner singularity, Boundary blow-up problem.

We consider several instances of the model semi-linear elliptic problem:

$$\Delta u = p(x)\,\mathrm{e}^u \text{ in } \Omega, \quad \text{with } p \geq 0 \text{ a.e. in } \Omega\,; \quad \text{plus boundary conditions on } \partial\Omega. \tag{1}$$

An example is the *Boltzmann–Poisson problem*:

$$-\Delta\phi = \kappa\,\mathrm{e}^{\phi_\mathrm{e}-\phi} := \rho, \quad \int_\Omega \rho\,\mathrm{d}x = M, \quad \phi = 0 \text{ on } \Gamma_1 \cup \Gamma_2, \quad \partial_\nu\phi = 0 \text{ on } \Gamma_3. \tag{2}$$

where $M > 0$ is given and $\kappa > 0$ is unknown. This system [1] models the equilibrium of charged particles in their self-consistent potential $\phi$ and an external potential $\phi_\mathrm{e} \in L^\infty(\Omega)$ solution to the linear problem:

$$-\Delta\phi_\mathrm{e} = \rho_\mathrm{e} \text{ in } \Omega, \quad \phi_\mathrm{e} = 0 \text{ on } \Gamma_1, \quad \phi_\mathrm{e} = \phi_\mathrm{in} \text{ on } \Gamma_2, \quad \partial_\nu\phi_\mathrm{e} = 0 \text{ on } \Gamma_3. \tag{3}$$

Above, we have set: $\partial\Omega = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3$, with $\Gamma_2$ and $\Gamma_3$ possibly empty. When $\Omega$ is a non-convex and non-smooth domain of $\mathbb{R}^d$, the potential $\phi$ is *singular*, i.e., $\phi \notin H^2(\Omega)$. In the case of a polygon in $\mathbb{R}^2$, the singular part of the solution is described by a finite number of singularity coefficients $\lambda$. We are interested in the asymptotic behaviour of the coefficients $\kappa,\ \lambda$ under various limits, such as $M \to +\infty$ and $\phi_\mathrm{in} \to +\infty$.

A *large* solution to (1) corresponds to a so-called *boundary blow-up problem*:

$$\Delta u = p(x)\,\mathrm{e}^u \text{ in } \Omega, \quad u \to +\infty \text{ near } \Gamma_B, \quad u = g_D \text{ on } \Gamma_D, \quad \partial_\nu u = g_N \text{ on } \Gamma_N, \tag{4}$$

where $\Gamma_D$ and $\Gamma_N$ can be empty, but: $\Gamma_D \neq \emptyset \implies \Gamma_N \neq \emptyset$ and $\overline{\Gamma}_B \cap \overline{\Gamma}_D = \emptyset$. The problem (4) has been studied by many authors in the case $\Gamma_B = \partial\Omega$; see [2] for a review. Under "reasonable" conditions, we show that it has a unique solution when $\Omega$ is a polygon.

Using large solutions as auxiliary functions, we answer the asymptotic questions on the Boltzmann–Poisson problem [1]. It is worth noting that the usual boundary layer techniques *do not work*, as the limiting problems seem to be ill-posed.

## References

[1] F. KARAMI, S. LABRUNIE, B. PINÇON. *Singularities of Stationary solutions to the Vlasov–Poisson System in a Polygon*, Math. Models Methods Appl. Sci. vol. 23, pp. 1029–1066, 2013.

[2] C. BANDLE, *Asymptotic behavior of large solutions of elliptic equations*, An. Univ. Craiova Ser. Mat. Inform., vol. 32, pp. 1–8, 2005.

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# Stationary 2D and 3D results for the Oseen and Navier-Stokes problem with singular data

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**Keywords:** Oseen equations, Navier-Stokes equations, very weak solutions, stationary solutions

We want to generalized the results in [3] about the existence of different types of solutions $(\boldsymbol{u}, \pi)$ in the 3-dimensional case for the Oseen and Navier-Stokes Problem. The aim is to treat the 2-dimensional case and improve the 3-dimensional one.

The Oseen problem (O) defined in a $\mathcal{C}^{1,1}$ domain $\Omega \subset \mathbb{R}^n$, $n = 2, 3$, is given by:

$$(O) \quad -\Delta \boldsymbol{u} + \boldsymbol{v} \cdot \nabla \boldsymbol{u} + \nabla \pi = \boldsymbol{f} \quad \text{and} \quad \nabla \cdot \boldsymbol{u} = h \quad \text{in } \Omega, \quad \boldsymbol{u} = \boldsymbol{g} \quad \text{on } \Gamma$$

assuming that $\boldsymbol{v} \in \mathbf{L}^n(\Omega)$. The 3-dimensional case, for a given solenoidal velocity $\boldsymbol{v}$ belonging to a space of type $L^s(\Omega)$ with $s$ depending on the regularity solution to obtain (very weak, weak, generalized or strong), was treated in [3]. Here, we also treat the non-solenoidal case for $\boldsymbol{v}$ (assuming regularity and smallness hypothesis on $\nabla \cdot \boldsymbol{v}$) and suppose that $\boldsymbol{v} \in \mathbf{L}^3(\Omega)$ for any type of solution. In this sense, the estimates are also improved, sometimes using the ideas in [2].

With respect to the 2-dimensional case, the main difficulty is the obtention of a solution for the Oseen problem in $\mathbf{H}^1(\Omega) \times L^2(\Omega)/\mathbb{R}$, where the arguments from [3] do not work and must be adapted using the results on [5] and [1].

By using a fixed-point argument, the existence of very weak solutions for the Navier-Stokes problem (NS), given by:

$$(NS) \quad -\Delta \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \nabla \pi = \boldsymbol{f} \quad \text{and} \quad \nabla \cdot \boldsymbol{u} = h \quad \text{in } \Omega, \quad \boldsymbol{u} = \boldsymbol{g} \quad \text{on } \Gamma,$$

will be obtained in the 2-dimensional and 3-dimensional case. Moreover, as a consequence of the study made for the Oseen problem, we can find solutions in the fractionary Sobolev spaces for (NS).

## References

[1] AMROUCHE, C., GIRAULT, V., *Decomposition of vector spaces and application to the Stokes problem in arbitrary dimension*, Czechoslovak Mathematical Journal **44** 119, 109–140 (1994).

[2] AMROUCHE, C., MESLAMI, M., ŠÁRKA NEČASOVÁ, *Uniqueness and regularity for the Oseen equations in a exterior domain*, In preparation.

[3] AMROUCHE, C. RODRÍGUEZ-BELLIDO, M. A., *Stationary Stokes, Oseen and Navier-Stokes equations with singular data*, Arch. Rational Mech. Anal. 199, 597–651 (2011).

[4] AMROUCHE, C. RODRÍGUEZ-BELLIDO, M. A., *Stationary 2D Stokes, Oseen and Navier-Stokes equations with singular data*, In preparation.

[5] COIFMAN, R., LIONS, P. L., MEYER, Y., SEMMES, S., *Compensated compactness and Hardy spaces*, J. Math. Pures Appl. **72**, 247–286 (1993).

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# Global spatial regularity for elasticity models with cracks and contact

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**Keywords:** global spatial regularity, cracks with selfcontact, Signorini contact with nonsmooth foundation, difference quotients, Tresca friction

A global higher differentiability result in Besov spaces is proved for the displacement fields of linear elastic models with self contact. In particular, domains with cracks are studied, where nonpenetration conditions/Signorini conditions are imposed on the crack faces. It is shown that in a neighborhood of crack tips (in 2d) or crack fronts (3d) the displacement fields are $B^{\frac{3}{2}}_{2,\infty}$-regular. The proof relies on a difference quotient argument for the directions tangential to the crack. In order to obtain the regularity estimates also in the normal direction, an argument due to Ebmeyer/Frehse/Kassmann is modified. The methods used here will also be applied to further examples like contact problems with nonsmooth rigid foundations or to energies with nonsmooth constraints as they occur for instance in the modeling of Tresca friction. Numerical examples will illustrate the proved results.

## References

[1] DOROTHEE KNEES, AMDREAS SCHRÖDER, *Global spatial regularity for elasticity models with cracks and contact*, Mathematical Methods in the Applied Sciences, Vol. 35(15), pp. 1859-1884, 2012.

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# Instance optimality of the adaptive maximum strategy

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**Keywords:** Adaptive finite element methods, optimal complexity, convergence, maximum marking strategy

Communication with reference [1].

### References

[1] LARS DIENING, CHRISTIAN KREUZER, ROB STEVENSON, *Instance optimality of the adaptive maximum strategy*, 2013, submitted, arXiv:1306.0377.

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# Dirichlet-to-Neumann boundary conditions for viscous acoustic equations in waveguides

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**Keywords:** Waveguide, Viscous Acoustic Equation, DtN

Let $\Omega \subset \mathbb{R}^2$ be a compact perturbation of a flat semi-infinite waveguide of width $L$ (see figure 1).

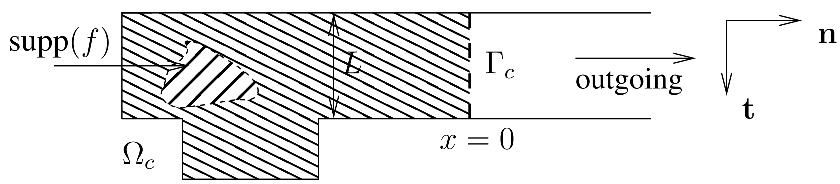


Figure 1: Perturbed waveguide $\Omega$

We consider time-harmonic acoustic velocity $\mathbf{v}$ and acoustic pressure $p$ (the time regime is $\exp(-\imath\omega t)$, $\omega > 0$) which are described by the coupled system in the framework of Landau and Lifshitz [1, 2]

$$\omega^2\rho_0\mathbf{v} + c_0^2\rho_0\nabla(\operatorname{div}\mathbf{v}) - \imath\omega\nu\Delta\mathbf{v} = \imath\omega\mathbf{f}, \text{ in } \Omega, \tag{1a}$$

$$-\imath\omega p + \rho_0 c_0^2 \operatorname{div}\mathbf{v} = 0, \text{ in } \Omega, \tag{1b}$$

$$\mathbf{v} = \mathbf{0}, \text{ on } \partial\Omega. \tag{1c}$$

Here we assume that $\mathbf{f} = 0$ on $\partial\Omega$ and $\mathbf{f}(x, y) = 0$ for $x \geq 0$. We can write an associated variational formulation on the bounded domain $\Omega_c$ with artificial boundary $\Gamma_c$ defined on figure 1, and we see that boundary terms contains $\operatorname{div}\mathbf{v}$, $\nu\operatorname{curl}_{2D}\mathbf{v}$, and both components of the test function. Our goal is to build an exact Dirichlet-to-Neumann (DtN) boundary condition that links both $\operatorname{div}\mathbf{v}$ and $\nu\operatorname{curl}_{2D}\mathbf{v}$ to both components of $\mathbf{v}$.

To do so, we introduce $(w_j)_{j\geq 0}$ the normalized eigenfunctions of the 1D Laplace-Beltrami operator $\partial_y^2$ with Neumann Boundary condition and $(w_j)_{j<0}$ their renormalized derivatives, and we decompose $\mathbf{v}$ over the Laplace-Beltrami basis. Such a decomposition allows us to write relations satisfied mode by mode, and then to write four DtN operators linking $\operatorname{div}\mathbf{v}(x = 0)$ and $\nu\operatorname{curl}_{2D}\mathbf{v}(x = 0)$ with both components of $\mathbf{v}$:

$$\operatorname{div}\mathbf{v}(x = 0) = \sum_{j\geq 0} D_{j,\mathbf{n}} \langle \mathbf{v}\cdot\mathbf{n}, w_j\rangle_{\Gamma_c} w_j - \sum_{j\geq 1} D_{j,\mathbf{t}} \langle \mathbf{v}\cdot\mathbf{t}, w_{-j}\rangle_{\Gamma_c} w_j \tag{2}$$

$$\nu\operatorname{curl}_{2D}\mathbf{v}(x = 0) = -\sum_{j\geq 1} R_{j,\mathbf{t}} \langle \mathbf{v}\cdot\mathbf{t}, w_{-j}\rangle w_{-j} + \sum_{j\geq 1} R_{j,\mathbf{n}} \langle \mathbf{v}\cdot\mathbf{n}, w_j\rangle_{\Gamma_c} w_{-j} \tag{3}$$

with $D_{j,\mathbf{n}}$, $D_{j,\mathbf{t}}$, $R_{j,\mathbf{n}}$ and $R_{j,\mathbf{t}}$ depending on $\nu$, $j$ and physical parameters, and define operators that are continuous from $\mathrm{L}^2(\Gamma_c)$ to itself. We illustrate these results by numerical experiments.

## References

[1] Landau, L. D. and Lifshitz, E. M.: Fluid Mechanics. Course of theoretical physics, Vol. 6. Pergamon press, New York, 1959.

[2] Schmidt, K. and Thöns-Zueva, A. *Asymptotic analysis for acoustics in viscous gases close to rigid walls.* Preprint 2012-4, Inst. f. Math., TU Berlin, 2012

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# Corner asymptotics of the magnetic potential in the eddy-current model

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**Keywords:** Corner asymptotics, eddy-current, quasi-dual fonction method

We describe the magnetic potential in the vicinity of a corner of a conducting body embedded in a dielectric medium in a bidimensional setting [1]. We make explicit the corner asymptotic expansion for this potential as the distance to the corner goes to zero. This expansion involves singular functions and singular coefficients. We introduce a method for the calculation of the singular functions near the corner. We extend the quasi-dual function method initiated in [2] to the case of resonances to compute the singular coefficients. Estimates for the convergence of this method are proven. We illustrate the theoretical results with finite element computations.
The specific non-standard feature of this problem lies in the structure of its singular functions: They have the form of series whose first terms are harmonic polynomials and further terms are genuine non-smooth functions generated by the piecewise constant zeroth order term of the operator.

### References

[1] DAUGE M., DULAR P., KRÄHENBÜHL L., PÉRON V., PERRUSSEL R., POIGNARD C., *Corner asymptotics of the magnetic potential in the eddy-current model*, INRIA RR 8204, 2013.

[2] COSTABEL M., DAUGE M., YOSIBASH Z., *A quasidual function method for extracting edge stress intensity functions*, SIAM Jour. Math. Anal., 35(5), 2004.

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# On spectrum of an elastic solid with cusps

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**Keywords:** anisotropic elasticity, cusp, spectrum

The spectral problem of anisotropic elasticity with traction-free boundary condition is considered in a cusped bounded domain. Near a cusp, it is given by triples $(x_1, x_2, x_3)$ such that $x_3^{-1-\gamma}(x_1, x_2) \in \omega$, where $\omega$ is a two-dimensional Lipschitz domain with a compact closure. The following result will be discussed along with some others. Let $\gamma = 1$, then there exists a threshold $\lambda_\dagger > 0$ explicitly expressed in terms of the elasticity constants and certain characteristics of the domain $\omega$; the continuous spectrum coincides with the half-line $[\lambda_\dagger, \infty)$, whereas the interval $[0, \lambda_\dagger)$ contains only the discrete spectrum. The results are obtained in collaboration with S. Nazarov.

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# Interactions between moderately close inclusions. The Neumann case and the Dirichlet case in the plane.


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**Keywords:** perforated domain, asymptotic expansion, Dirichlet boundary conditions, Neumann boudary conditions


The presence of small inclusions or of a surface defect modifies the solution of the Laplace equation posed in a reference domain $\Omega_0$. If the characteristic size of the perturbation is small, then one can expect that the solution of the problem posed on the perturbed geometry is close to the solution of the reference shape. Asymptotic expansion with respect to that small parameter  the characteristic size of the perturbation  can then be performed. I will consider in the presentation first the case of two defects with homogeneous Neumann condition, then in a second step the case of two circular defects with homogeneous Dirichlet boundary conditions in a bidimensional domain. I will distinguish the cases where the distance between the object is of order 1 and the case where it is larger than the characteristic size of the defects but small with respect to the size of the domain:

- In the first case, we are considering two small holes of size $\varepsilon$ around two fixed points $A$ and $B$ and the distance between both is hence fixed. For the cases of Neumann boundary condition or of Dirichlet boundary conditions in dimension at least three, this cases can be treated by separating each hole through cut-off functions and hence reducing it to the single inclusion case (see [2]). For the Dirichlet case in the plane, the presence of the logarithmic term prohibits this approach and the interaction between the holes has to be studied.
- In the second case, the distance between $A$ and $B$ collapses to 0 with $\varepsilon$ like $\varepsilon^{\alpha}$, that is to say slower than the size $\varepsilon$ of the inclusions. The interaction between the two holes are then stronger and we will prove that the leading order of the asymptotic expansion is then modified (see[1]).

In both cases, we derive the complete expansion and provide some numerical illustrations.

### References


[1] V. BONNAILLIE-NOËL AND M. DAMBRINE, *Interactions between moderately close circular inclusions: The DirichletLaplace equation in the plane*, Asymptotic Analysis, accepted, 2013.

[2] V. BONNAILLIE-NOËL, M. DAMBRINE, S. TORDEUX AND G. VIAL, *Interactions between moderately close inclusions for the Laplace equation*, Math. Models Methods Appl. Sci. 19(10) (2009), 18531882.



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# Central module

# Maxwell's Equations & Relatives: The Structure of Electromagnetic Wave Propagation.

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**Keywords:** Evolutionary equations, coupled systems, Maxwell equations

Maxwell's equations will be embedded into a general class of well-posed linear evolutionary problems, which turns out to be spacious enough for the discussion of coupling phenomena of electromagnetic waves with other (linearized) phenomena (multiphysics). It is demonstrated that these problems share a common structure, which is in the simplest case of the form

$$(\partial_0 M_0 + M_1 + A)\, U = F,$$

where $\partial_0$ denotes the time-derivative, $A$ is a skew-selfadjoint operator and $M_0$, $M_1$ are suitable bounded operators in a Hilbert space $H$. The complexity of physical phenomena appear as encoded in the corresponding material law

$$M\left(\partial_0^{-1}\right) = M_0 + \partial_0^{-1} M_1.$$

The usefulness of this structural perspective is illustrated by applications to various coupled systems.

## References

[1] R. Picard and D. F. McGhee. *Partial Differential Equations: A unified Hilbert Space Approach*, volume 55 of *De Gruyter Expositions in Mathematics*. De Gruyter. Berlin, New York. 518 p., 2011.

[2] R. Picard. Mother Operators and their Descendants, Technical report, TU Dresden, arXiv:1203.6762v2, 2012. In press: JMAA (2013), doi:10.1016/j.jmaa2013.02.004

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# Diffraction from polygonal-conical screens - an operator approach

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**Roland DUDUCHAVA**, Tbilisi State University, Georgia

**Keywords:** Diffraction, plane screen, polygonal domain, conical domain, Dirichlet problem, Neumann problem, explicit solution.

The aim of this contribution is to construct explicitly resolvent operators for a class of boundary value problems in diffraction theory. These are formulated as boundary value problems for the three-dimensional Helmholtz equation with Dirichlet or Neumann conditions on a plane screen of polynomial-conical form (including unbounded and multiple-connected screens, see the definition below), in weak formulation. Briefly written

$$\begin{array}{lllll} Au & = & \left(\Delta + k^2\right) u & = & 0 \quad \text{in} \quad \Omega \\ Bu & & & = & g \quad \text{on} \quad \Gamma = \partial\Omega. \end{array}$$

Herein $k$ is the wave number and we assume that $\Im m\, k > 0$. $B$ stands for the boundary operator, taking the trace or normal derivative of $u$ on $\Gamma$. Hence the operator associated with the boundary value problem can be written as

$$B_0 \quad = \quad B|_{\ker A} \quad : \quad \mathcal{H}^1(\Omega) \to H^{\pm 1/2}(\Sigma)$$

where

$$\begin{array}{l} \Omega = \mathbb{R}^3 \setminus \Gamma \\ \Gamma = \overline{\Sigma} \times 0 = \{x = (x_1, x_2, 0) \in \mathbb{R}^3 : x' = (x_1, x_2) \in \overline{\Sigma}\}. \end{array}$$

$\mathcal{H}^1(\Omega)$ denotes the space of weak solutions of the Helmholtz equation in $\Omega$ and $B_0$ denotes the restriction of $B$ to this space. We are looking for the inverse $B_0^{-1}$, the so-called resolvent operator, in explicit form. A convex polygonal-conical domain (convex PCD) in $\mathbb{R}^2$ is given by

$$\Sigma \quad = \quad \bigcap_{j=1,\ldots,m} \Sigma_j \quad \text{where} \quad \Sigma_j \quad \text{are open half-planes.}$$

A polygonal-conical domain (PCD) in $\mathbb{R}^2$ is given by

$$\Sigma \quad = \quad \text{int} \bigcup_{j=1,\ldots,m} \text{clos}\, \Sigma_j \quad \text{where} \quad \Sigma_j \quad \text{are convex PCDs.}$$

**Main Theorem.** Let $\Sigma$ be a PCD. Then the resolvent operator $B_0^{-1}$ for the Dirichlet or Neumann problem is explicitly given in terms of infinite operator products which strongly converge in the common (Bessel potential) norm of $H^{\pm 1/2}(\mathbb{R}^2)$ for $k = i$ and in a modified equivalent norm for $k \in i\mathbb{R}_+$, respectively. In the remaining cases of $k \in \mathbb{C}$, $\Im m k > 0$ the resolvent operator can be explicitly represented by (additional) use of Neumann series.

The proof is based upon operator theoretical techniques in Hilbert spaces, such as the construction of matrical coupling relations and certain orthogonal projections, which represent new techniques in this area of applications. Various cross connections are used, particularly considering classical Wiener-Hopf operators in Sobolev spaces as general Wiener-Hopf operators in Hilbert spaces and studying relations between the crucial operators in game. In this way, former results of M. Costabel, E. Meister, F. Speck, E. Stephan et al. are extended, particularly to multi-connected screens.

The talk is based upon recent work with Luís Castro (Aveiro, Portugal) and Roland Duduchava (Tbilisi, Georgia).

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# Small heterogeneities in the context of time domain wave equation

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**Keywords:** Matched Asymptotic Expansions, Time domain Wave equations, Small heterogeneities

In this talk, we will present a way to compute the field scattered by a small obstacle with low computation burden based on the matched asymptotic expansions.

## 1 The considered problem

Let us consider a small obstacle $B_\varepsilon$ equipped with Dirichlet boundary conditions :

$$B_\varepsilon = \varepsilon\hat{B} = \Big\{(x,y,z) : \Big(\frac{x}{\varepsilon},\frac{y}{\varepsilon},\frac{z}{\varepsilon}\Big) \in \hat{B}\Big\}, \tag{1}$$

with $\hat{B}$ a reference shape and $\partial B_\varepsilon = \varepsilon\partial\hat{B}$ its boundary. The propagation domain $\Omega_\varepsilon$ consists of the exterior to the obstacle $B_\varepsilon$ :

$$\Omega_\varepsilon = \mathbb{R}^3 \backslash B_\varepsilon. \tag{2}$$

We denote by $f \in \mathcal{D}\left(\mathbb{R}^3 \times \overline{\mathbb{R}_+}\right)$ a source term satisfying : there exists $\varepsilon_0 > 0$ such that

$$f(\mathbf{x},t) = 0, \quad \text{for } \|\mathbf{x}\| < \varepsilon_0 \text{ and } t \geq 0. \tag{3}$$

Let us consider the solution of the $3D$ time-domain wave equation :

$$\frac{\partial^2 u_\varepsilon}{\partial t^2}(\mathbf{x},t) - \Delta u_\varepsilon(\mathbf{x},t) = f(\mathbf{x},t), \quad \mathbf{x} \in \Omega_\varepsilon,\ t \geq 0,$$

equipped with the Dirichlet boundary condition :

$$u_\varepsilon(\mathbf{x},t) = 0, \quad \mathbf{x} \in \partial\Omega_\varepsilon,\ t > 0 \tag{4}$$

and homogeneous initial conditions :

$$u_\varepsilon(\mathbf{x},0) = 0, \quad \partial_t u_\varepsilon(\mathbf{x},0) = 0. \tag{5}$$

**Remark**. For the sake of simplicity we have assumed that the wave speed is equal to 1.

## 2 Matching of asymptotic expansions

The matching of asymptotic expansions [1] is an asymptotic domain decomposition method with overlapping. It consists in representing the solution with a far-field expansion far away from the obstacle and a near-field expansion near the obstacle. These two expansions are matched in a transition zone with the so-called Van Dyke matching conditions. This approach is equivalent to the corrector method [2, 3].

### 2.1 The far-field expansion

The far-field expansion is defined on the far-field domain $\Omega_* = \mathbb{R}^3 \setminus \{\mathbf{0}\}$ consisting of the limit of $\Omega_\varepsilon$ for $\varepsilon$ varying to 0. It takes the form of a Taylor series :

$$u_{\varepsilon,I}(\mathbf{x},t) = \sum_{i=0}^{I} u_i(\mathbf{x},t)\varepsilon^i. \tag{6}$$

The first term of this expansion $u_0 : \mathbb{R}^3 \longrightarrow \mathbb{R}$ is the limit of $u_\varepsilon$ for $\varepsilon$ varying to 0. It is a regular solution over all $\mathbb{R}^3$ of the time-domain wave equation :

$$\frac{\partial^2 u_0}{\partial t^2}(\mathbf{x},t) - \Delta u_0(\mathbf{x},t) = f(\mathbf{x},t), \quad \mathbf{x} \in \mathbb{R}^3,\ t > 0,$$

equipped with the initial conditions :

$$u_0(\mathbf{x},0) = 0, \quad \partial_t u_0(\mathbf{x},0) = 0, \quad \mathbf{x} \in \mathbb{R}^3. \tag{7}$$

The next coefficients $u_i : \Omega_* \longrightarrow \mathbb{R}$ of this expansion are solutions of the homogeneous time-domain wave equation :

$$\begin{cases} \dfrac{\partial^2 u_i}{\partial t^2}(\mathbf{x},t) - \Delta u_i(\mathbf{x},t) = 0, & \mathbf{x} \in \Omega_*,\ t > 0, \\ u_i(\mathbf{x},0) = 0, \quad \partial_t u_i(\mathbf{x},0) = 0, & \mathbf{x} \in \Omega_*, \end{cases}$$

which are singular in the neighbourhood of $\mathbf{x} = 0$. This power series aims at approximating the solution $u_\varepsilon$ at fixed $\mathbf{x} \neq \mathbf{0}$ :

$$u_\varepsilon(\mathbf{x},t) - u_{\varepsilon,I}(\mathbf{x},t) = \underset{\varepsilon \to 0}{O}(\varepsilon^{I+1}). \tag{8}$$

In the case of a spherical obstacle, $B_\varepsilon = \{\mathbf{x} \in \mathbb{R}^3 : \|\mathbf{x}\| \leq \varepsilon\}$, the second order far-field expansion is given by :

$$u_{\varepsilon,2}(\mathbf{x},t) = u_0(\mathbf{x},t) - \varepsilon \frac{u_0(0,t-t_0)}{R} - \varepsilon^2 \left( \frac{\partial_t u_0(0,t-t_0)}{R} \right),$$

where $t_0 = \frac{R}{c}$ and $R = \sqrt{x^2+y^2+z^2}$.

### 2.2 The near-field expansion

The near-field domain $\widehat{\Omega}$ consists in the normalization of the original domain $\Omega_\varepsilon$ :

$$\widehat{\Omega} = \frac{\Omega_\varepsilon}{\varepsilon} = \{(X,Y,Z) \in \mathbb{R}^3 : \varepsilon X, \varepsilon Y, \varepsilon Z \in \Omega_\varepsilon\} \tag{9}$$

The near-field expansion takes the form :

$$\sum_{i=0}^{+\infty} U_i(\mathbf{x},t)\varepsilon^i, \tag{10}$$

which aims at approximating $U_\varepsilon(\mathbf{x},t) = u_\varepsilon(\varepsilon\mathbf{x},t)$ at fixed $\mathbf{x} \in \widehat{\Omega}$. The coefficients of the near field expansion satisfy the hierarchical Laplace equation :

$$\Delta U_i(\mathbf{x},t) = \partial_t^2 U_{i-2}(\mathbf{x},t), \quad \mathbf{x} \in \widehat{\Omega},\ t > 0, \tag{11}$$

equipped with the Dirichlet boundary condition :

$$U_i(\mathbf{x},t) = 0, \quad \mathbf{x} \in \partial\widehat{\Omega}, \tag{12}$$

where we have used the convention $U_i \equiv 0$ for $i < 0$.

## 3 Numerical results

We will also present the results of a numerical experiment. We compare the second order far-field expansion to a direct numerical approximation of $u_\varepsilon$ achieved with an Interior Penalty Discontinuous Galerkin Method (IPDG) associated to a second order BGT absorbing boundary condition and to a local space-time mesh refinement [4].

**References**


[1] A. M. Il'in, *Matching of asymptotic expansions of solutions of boundary value problems*, American Mathematical Society, 1992.

[2] V. Maz'ya, S. Nazarov and B. Plamenevskij, *Asymptotic theory of elliptic boundary value problems in singularly perturbed domains*, Birkhuser Verlag, Basel . Boston . Berlin, 2000.

[3] Gabriel Caloz, Martin Costabel, Monique Dauge and Grégory Vial Asymptotic expansion of the solution of an interface problem in a polygonal domain with thin layer . Asymptotic Analysis 50 (2006), 121173.

[4] J. Diaz and M. Grote *Energy conserving explicit local time stepping for second-order wave equations* SIAM J. SCI. COMPUT., vol. 31, No. 3, pp. 1985-2014.

[5] S. Tordeux, *Méthodes asymptotiques pour la propagation des ondes dans les milieux comportant des fentes* Thèse de l'Université de Versailles, 2004.

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# Multilayer shallow water modeling for free surface flows

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**Keywords:** Multilayer models, shallow water, polydisperse sedimentation, Navier-Stokes equations, finite volumes methods, fluid dynamics, free surface flows

Multilayer Saint-Venant (or shallow water) models are commonly used techniques to study hydrodinamic flows with large friction coefficients, with significant water depths and/or with important wind effects, among others (see for example [3], [4], [5]). In these cases, the standard shallow water system is controversial since the horizontal velocity can hardly be approximated by a vertically constant velocity in the whole domain. The multilayer approach consists in subdividing, in the vertical direction, the domain into shallow layers in order to apply inside the classic hypothesis of Saint-Venant. That produces a vertically varying profil. First, we apply a standard multilayer approach to study polydisperse sedimentation. Actually we consider the flow of a fluid containing one disperse substance consisting of small particles that belong to different species differing in size and density. The polydisperse sedimentation model derived in [6], for a single layer, is used here as the reference model. Next, we develop a new multilayer technique based on a particular approximation of the weak formulation of the partial differential equations governing the flow. For incompressible fluids, this technique yields a piecewise linear vertical velocity while the standard approach leads to a piecewise constant profil which, in addition, is computed by a postprocessing. The final models obtained from these approaches are all written under the structure of an hyperbolic system with conservative components, nonconservative products and source terms. We then resolve them using finite volume schemes. More precisely, we exploit PVM (Polynomial Viscosity Matrix) methods which constitute a class of fast finite volume solvers for balance laws or, more generally, for nonconservative hyperbolic systems (see [7]). They are defined in terms of viscosity matrices computed by a suitable polynomial evaluation of a Roe matrix. These methods have the advantage that they only need some information about the eigenvalues of the system to be defined and no spectral decomposition of Roe matrix is necessary. As a consequence, they are faster than Roe method. PVM methods may be seen as a generalization of various classic schemes in the literature.

### References

[1] E. FERNÁNDEZ-NIETO, E. H. KONÉ, T. MORALES and R. BÜRGER, *A multilayer shallow water system for polydisperse sedimentation*, J. Comput. Phys., **238**, pp 281-314, 2013.

[2] E. FERNÁNDEZ-NIETO, E. H. KONÉ and T. CHACÓN, *A multilayer approach method for 3D models: A particular weak solution*, submitted in january 2013.

[3] E. AUDUSSE, *A multilayer Saint-Venant model: derivation and numerical validation*, Discrete Contin. Dyn. Syst. Ser. B., **5(2)**, pp 189-214, 2005.

[4] E. AUDUSSE, M.-O. BRISTEAU, B. PERTHAME and J. SAINTE-MARIE, *A multilayer Saint-Venant system with mass exchanges for shallow water flows. Derivation and numerical validation*, ESAIM Math. Model. Numer. Anal., **45(1)**, pp 169-200, 2011.

[5] J. SAINTE-MARIE, *Vertically averaged models for the free surface non-hydrostatic Euler system: derivation and kinetic interpretation*, Math. Models Methods Appl. Sci., **21(3)**, pp 459-490, 2011.

[6] S. BERRES, R. BÜRGER, K.H. KARLSEN and E.M. TORY, *Strongly degenerate parabolichyperbolic systems modeling polydisperse sedimentation with compression*, SIAM J. Appl. Math., **64(1)**, pp 41-80, 2003.

[7] M. J. CASTRO DÍAZ and E. FERNÁNDEZ-NIETO, *A class of computationally fast first order finite volume solvers: PVM methods*, SIAM J. Sci. Comput., **34(4)**, pp 2173-2196, 2012.

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# Invisible deformation of the boundary of an acoustic waveguide

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**Keywords:** waveguide, scattering matrix, invisibility, asymptotic theory, fixed point

The question of whether a scatterer can be invisible, in the sense that it does not scatter incident waves, is currently the subject of an active research. Here we consider a two-dimensional homogeneous acoustic waveguide and we aim at designing deformations of the boundary which are invisible at a given frequency, or more generally at a finite number of given frequencies. To find such invisible perturbations, we take advantage of the fact that there are only a finite number of propagative modes at a given frequency in a waveguide. As a consequence, the invisibility is achieved by canceling a finite number of scattering coefficients, and an invisible deformation only produces an exponentially decreasing scattered field, not measurable in the far field.
The first step consists in studying the effect of a small deformation, of amplitude $\varepsilon$. The asymptotic analysis allows to derive the first order terms of the scattering coefficients, as integrals involving the function describing the deformation. For instance, les us consider a waveguide of height 1 and a wavenumber $k < \pi$, so that only the plane mode is propagating. Then a small deformation of the upper boundary given by $y = \varepsilon h(x)$ produces a reflection of the plane mode, with a reflection coefficient $R_\varepsilon$ such that

$$R_\varepsilon = ik\varepsilon \int h(x)e^{2ikx}dx + O(\varepsilon^2).$$

This leads, following [1] (where the approach of [3] is used), to express the deformation $h$ as a linear combination of some explicit (compactly supported) functions, so that the condition $R_\varepsilon = 0$ is satisfied if and only if the coefficients of the linear combination are solution of a fixed point equation. The key point is that we can prove, using the results of the asymptotic analysis, that the function of this fixed point equation is a contraction for $\varepsilon$ small enough. This proves the existence of invisible deformations of amplitude $\varepsilon$ (only a phase shift of order $\varepsilon^2$ remains that we are not able to remove). Moreover, it provides a natural algorithm to compute the invisible deformation.
This has been tested numerically. At each iteration of the fixed-point algorithm, we have to compute $R_\varepsilon$ for a given $h$. To avoid remeshing, we use a multimodal method as in [2]. The results are in perfect agreement with the theory. The good news is that $\varepsilon$ can be taken quite large (the amplitude of the deformation may be half the size of the guide). We are able to build both invisible bumps and invisible cavities. Let us point out that an invisible cavity leads by symmetry to an invisible obstacle in the middle of a guide two times larger.

## References

[1] A.-S. BONNET–BEN DHIA, S.A. NAZAROV, *Obstacles in acoustic waveguides becoming "invisible" at given frequencies.* , Acoustical physics, to appear.

[2] C. HAZARD AND E. LUNÉVILLE, *An improved multimodal approach for non uniform acoustic waveguides*, IMA Journal of Applied Math., 73 (4), pp. 668–690, 2008.

[3] S.A. NAZAROV, *Asymptotic expansions of eigenvalues in the continuous spectrum of a regularly perturbed quantum waveguide*, Theoretical and Mathematical Physics, 167 (2), pp. 606–627, 2011.

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# Numerical approximation of transmission problems with sign changing coefficients

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**Keywords:** wave transmission problem, interface problem, meta-materials, T-coercivity, Finite Element Method

Some electromagnetic materials exhibit, in a given frequency range, effective dielectric permittivity and/or magnetic permeability which are negative when dissipation is neglected. In the literature, they are called negative index materials, left-handed materials or meta- materials. In this talk, we review some numerical methods that allow one to solve wave transmission problems between a classical dielectric material and a meta-material. Because of the sign-change of the coefficient, the resulting problem is indefinite.

At the continuous level, the solution methods are based on the so-called $T$-coercivity approach, which amounts to the use of explicit inf-sup operators, as introduced in [2]. In this Ref. the explicit operators $T$ are based on the lifting of traces on the interface that separates the classical material and the meta-material. Then, one can try and adapt this approach to the discrete problems, discretized by a conforming Finite Element Method ($P_k$ Lagrange FE, $k \geq 1$). A first instance has been given in [2], which was further refined in [5]. In the same spirit, one can use staggered discontinuous Galerkin finite elements to solve numerically the same problem [4].

On the other hand, the technique was recently revisited at the continuous level, see [1]. In this Ref., the authors proposed another family of explicit operators $T$, which relies on local elementary geometrical mappings (symmetries, rotations), combined to a localization argument. Then, the discrete problems (obtained via a conforming FEM) were solved in [3], in which the notion of locally T-conform meshes was introduced. More precisely, one can prove convergence of the discrete solution the the exact solution under the following condition: the discrete space on one side of the interface is equal to the discrete space on the other side, transformed by the operator $T$. Provided this (sufficient) condition holds in a neighborhood of the interface, convergence follows. Among others, we propose numerical evidence that show that this sufficient condition is necessary in some sense. Finally, we study how one can solve eigenvalue problems that involve an interface between a classical dielectric material and a meta-material.

## References

[1] A.-S. BONNET-BEN DHIA, L. CHESNEL, P. CIARLET, JR., *T-coercivity for scalar interface problems between dielectrics and metamaterials*, Math. Mod. Num. Anal., 46, pp. 1363–1387, 2012.

[2] A.-S. BONNET-BEN DHIA, P. CIARLET, JR., C.M. ZWÖLF, *Time harmonic wave diffraction problems in materials with sign-shifting coefficients*, J. Comput. Appl. Math., 234, pp. 1912–1919, 2010.

[3] L. CHESNEL, P. CIARLET, JR., *T-coercivity and continuous Galerkin methods: application to transmission problems with sign changing coefficients*, Numer. Math., 124, pp. 1–29, 2013.

[4] E.T. CHUNG, P. CIARLET, JR., *Scalar transmission problems between dielectrics and metamaterials: T-coercivity for the Discontinuous Galerkin approach*, J. Comput. Appl. Math., 239, pp. 189–207, 2013.

[5] S. NICAISE, J. VENEL, *A posteriori error estimates for a finite element approximation of transmission problems with sign changing coefficients*, J. Comput. Appl. Math., 235, pp. 4272–4282, 2011.

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# Mapping properties of retarded potentials and applications to a posteriori estimates

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**Keywords:** time-dependent boundary integral operators, boundary element method, a posteriori estimates

Consider a boundary integral formulation for the 3–dimensional wave equation with Dirichlet or acoustic boundary conditions outside a Lipschitz domain $\Omega$. We make first steps to analyze its adaptive Galerkin approximation by the time–dependent boundary element method using analytical convolution in time. Similar as for elliptic problems [1, 2], mapping properties of the relevant boundary integral operators on $\partial\Omega$ in the scale of (space–time anisotropic) Sobolev spaces are crucial for an a posteriori error analysis. We discuss the relevant operator estimates, corresponding to Costabel's results for the elliptic case [3], and use them to study a posteriori error estimates for the numerical solutions. Also computational aspects will be considered.

## References

[1] C. CARSTENSEN, *Efficiency of a posteriori BEM-error estimates for first-kind integral equations on quasi-uniform meshes*, Math. Comp. **65** (1996), 69-84.

[2] C. CARSTENSEN AND E. P. STEPHAN, *A posteriori error estimates for boundary element methods*, Math. Comp. **64** (1995), 483–500.

[3] M. COSTABEL, *Boundary integral operators on Lipschitz domains: elementary results*, SIAM J. Math. Anal. **19** (1988), 613–626.

[4] M. COSTABEL, *Time-dependent problems with the boundary integral equation method*, John Wiley and Sons (2004).

[5] H. GIMPERLEIN, Z. NEZHI AND E. P. STEPHAN, *Mapping properties of retarded potentials and applications to a posteriori estimates*, in preparation (2013).

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# Shape reconstruction of non-convex elastic scatterers using a regularized Newton-type method

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**Keywords:** fluid-solid interaction, scattering problem, elastic scatterer, inverse obstacle problem, ill-posed problem, Fréchet derivative, Newton method, Tikhonov regularization, finite element method

The determination of the shape of an obstacle from its effects on known acoustic or electromagnetic waves is an important problem in many technologies such as sonar, geophysical exploration and medical imaging or non-destructive testing. Because of its ill-posed and nonlinear character, this inverse obstacle problem (IOP) is difficult to solve, especially from a numerical viewpoint. Any attempt to its investigation requires the fundamental understanding of the theory for the associated direct scattering problem, and the mastery of the corresponding numerical solution methods.

In this work, we are interested in retrieving the shape of an elastic scatterer from the knowledge of some scattered far-field patterns, and assuming certain characteristics of the surface of the obstacle. We propose a solution methodology based on a regularized Newton-type method, known to be robust and efficient to solve this class of IOPs. Note that this approach can be viewed as an extension to the method proposed in [1] for the case of impenetrable scatterers. The solution of this IOP by a regularized Newton method incurs, at each iteration, the solution of a linear system whose entries are the Fréchet derivatives of the elasto-acoustic field with respect to the shape parameters. Moreover, ensuring the stability, fast convergence, and computational efficiency calls for computing these derivatives with a greater robustness and a higher level of accuracy than possible with finite differences. To this effect, following the approach used in [2] for the case of exterior Helmholtz problems, we have characterized the Fréchet derivative of the elasto-acoustic scattered field with respect to the shape of an elastic obstacle as the solution of a direct elasto-acoustic scattering problem which differs from the considered direct scattering problem only in the transmission conditions on the surface of the scatterer [3]. It is worth noting that this characterization requires an elaborated and careful work to give a sense to the transmission conditions on the boundary of the obstacle, since this work depends on the smoothness of the surface of the scatterer and involves the use of nonstandard trace theorems. Consequently, at each regularized Newton iteration, we can thus evaluate both the scattered field and the directional derivatives by solving a single system of equations with different right-hand sides.

Clearly, the performance of the IOP solver depends mainly on the computational efficiency of the solution of the forward problems that arise at each Newton iteration. To this end, we propose to solve the direct scattering-type problems using a finite-element method based on discontinuous Galerkin approximations, equipped with curved boundary edges [4]. We anticipate to present numerical results that illustrate the potential of the proposed solution methodology for retrieving the shape of scatterers with some singularities.

### References

[1] R. DJELLOULI, C. FARHAT, R. TEZAUR, *On the solution of three-dimensional inverse obstacle acoustic scattering problems by a regularized Newton method*, Inverse Problems,18, 1229-1246, 2002.

[2] R. DJELLOULI, C. FARHAT, *On the Characterization of the Fréchet Derivative with Respect to a Lipschitz Domain of the Acoustic Scattered Field*, J. Math. Anal. Appl., 238, 259-276, 1999.

[3] H. BARUCQ, R. DJELLOULI, E. ESTECAHANDY, *Characterization of the Fréchet derivative of the elasto-acoustic field with respect to Lipschitz domains*, accepted, 2013.

[4] H. BARUCQ, R. DJELLOULI, E. ESTECAHANDY, *Efficient DG-like formulation equipped with curved boundary edges for solving elasto-acoustic scattering problems*, submitted, 2013.

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# On the Maxwell Constants in 3D

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**Keywords:** Maxwell constants, Maxwell estimates, Poincaré constants, Poincaré estimates

We prove that for bounded and convex domains in 3D, the Maxwell constants corresponding to homogeneous tangential or normal boundary conditions are bounded from below and above by the Poincaré constants for the Dirichlet resp. the Neumann-Laplace problem. In other words, the second Maxwell eigenvalue lies between the second Neumann-Laplace and the first Dirichlet-Laplace eigenvalue.

## References

[1] AMROUCHE, C. AND BERNARDI, C. AND DAUGE, M. AND GIRAULT, V., *Vector potentials in three-dimensional non-smooth domains*, Math. Methods Appl. Sci., 21(9), 823-864, 1998.

[2] COSTABEL, M., *A coercive bilinear form for Maxwell's equations*, J. Math. Anal. Appl., 157(2), 527-541, 1991.

[3] LEIS, R., *Initial Boundary Value Problems in Mathematical Physics*, Teubner, Stuttgart, 1986.

[4] SARANEN, J., *On an inequality of Friedrichs*, Math. Scand., 51(2), 310-322, 1982.

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# Time Domain Electromagnetic Scattering from a Penetrable Medium

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**Keywords:** Electromagnetic scattering, time domain, integral equations, convolution quadrature

We shall show that the classical Convolution Quadrature method based on multistep A-stable ordinary differential equation solver [1] applied to a boundary integral formulation of the time domain Maxwell system governing waves scattered by a homogeneous penetrable isotropic object results in a convergent scheme. In particular we follow Laliena and Sayas [2] and relate the integral equation system to a generalized transmission problem for Maxwell's equations. In addition we shall present some numerical results computed using the time domain toolbox within the BEM++ library[3]

### References

[1] C. Lubich, *On the multistep time discretization of linear initial-boundary value problems and their boundary integral equations*, Numer. Math., 67 (1994), pp. 365389.

[2] A. Laliena and F. Sayas, *Theoretical aspects of the application of convolution quadrature to scattering of acoustic waves*, Numer. Math., 112 (2009), pp. 637-678.

[3] W. Śmigaj, S. Arridge, T. Betcke, J. Phillips, M. Schweiger, *Solving Boundary Integral Problems with BEM++*, submitted to ACM Trans. Math. Software.

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# A Multigrid Method for Two-dimensional Maxwell Interface Problems

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**Keywords:** Maxwell interface problems, multigrid

It is well-known that the solutions of Maxwell interface problems can have very low regularity and are therefore difficult to capture numerically. In this talk I will discuss a multigrid method for such problems that is based on (i) a reduction of two dimensional time harmonic Maxwell interface problems to interface problems for the Laplace operator and (ii) information on the singularities of interface problems for Laplace operators.

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# Tensor product finite element differential forms

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**Keywords:** Finite element exterior calculus, Approximation properties, Quadrilaterals

In this communication we discuss the approximation properties of finite elements constructed using a tensor product technique.
It is well-known that standard finite elements may suffer from suboptimal order of approximation on general meshes of distorted quadrilaterals or hexahedra [1], [2].
In [3] the construction of tensor product finite element differential forms in the framework of finite element exterior calculus is discussed. This setting allows the general construction of finite elements in any space dimensions and for any differential form degree. We denote the general reference element by $\mathcal{Q}_r^-\Lambda^k(I^n)$, where $r$ is the polynomial degree, $k$ the form degree, $n$ the space dimension, and $I$ the unit reference interval. Starting from shape functions defined on the reference element $I^n$, the actual finite element space is defined as usual via the pullback of the diffeomorphism mapping the reference element into the physical one. In three dimensions, this construction gives the standard cubic finite elements approximating the spaces $H^1(\Omega)$, $H(\mathbf{curl};\Omega)$, $H(\mathrm{div};\Omega)$, and $L^2(\Omega)$.
In this paper we discuss the approximation properties of the tensor product finite element space $\mathcal{Q}_r^-\Lambda^k$. It turns out that when the maps from the reference cube are affine, the approximation rate depends only on the degree of polynomials contained in the reference space; in the more general case, when the maps are multilinear, a degradation in the approximation rate is observed, the loss being more severe for higher degree differential forms.

### References

[1] DOUGLAS N. ARNOLD, DANIELE BOFFI, RICHARD S. FALK, *Approximation by quadrilateral finite elements*, Math. Comp., 71(239), 909–922, 2002.

[2] DOUGLAS N. ARNOLD, DANIELE BOFFI, RICHARD S. FALK, *Quadrilateral H(div) finite elements*, SIAM J. Numer. Anal., 42(6), 2429–2451, 2005.

[3] DOUGLAS N. ARNOLD, DANIELE BOFFI, FRANCESCA BONIZZONI, *Tensor product finite element differential forms and their approximation properties*, arXiv preprint 1212.6559, 2012.

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# Contraction estimates of the double layer integral operator and applications

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**Keywords:** Double layer potential, contraction, preconditioning, FEM/BEM coupling

The proof of the contraction property of the double layer boundary integral operator $\frac{1}{2}I+K : H^{1/2}(\Gamma) \to H^{1/2}(\Gamma)$ is based on the use of appropriate Sobolev norms and different representations of the Steklov–Poincaré operators, see [4], or equivalently, on energies which are related to interior and exterior boundary value problems, see [1]. Besides a review on previous results we will discuss an alternative characterisation by considering related Steklov eigenvalue problems, which also allows to describe the involved system of orthogonal eigenfunctions. In addition we also comment on the numerical approximation of the contraction constant $c_K < 1$.
Applications include boundary integral operator preconditioners of opposite order [3] and the stability of coupled finite and boundary element formulations [2].

## References

[1] M. COSTABEL, *Some historical remarks on the positivity of boundary integral operators.* In: Boundary element analysis, pp. 1-27, Lect. Notes Appl. Comput. Mech., vol. 29, Springer, Berlin, 2007.

[2] O. STEINBACH, *A note on the stable one–equation coupling of finite and boundary elements*, SIAM J. Numer. Anal. 49 (2011) 1521–1531.

[3] O. STEINBACH, W. L. WENDLAND, *The construction of some efficient preconditioners in the boundary element method*, Adv. Comput. Math. 9 (1998) 191–216.

[4] O. STEINBACH, W. L. WENDLAND, *On C. Neumann's method for second order elliptic systems in domains with non–smooth boundaries*, J. Math. Anal. Appl. 262 (2001) 733–748.

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# Riesz potentials and quasi-local transmission condition for iterative non overlapping domain decomposition methods for the Helmholtz equation

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**Keywords:** Helmholtz, Domain decomposition, Riesz potentials

In this work, we address the question of iterative non overlapping domain decomposition methods for the Helmholtz equation, in the spirit of [1]. Our main objective is to propose novel transmission conditions, with respect to existing works, in order to improve the convergence rate of such methods. We use impedance based condition transmission. In the simple case where the domain $\Omega$ (in which the original problem is posed) is splitted into two subdomains $\Omega_1$ and $\Omega_2$ separated by a closed interface $\Sigma$, we introduce, for any $\lambda \in \mathbb{C}$, the trace operator

$$B_\lambda := \partial_n + \lambda T$$

where $T \in \mathcal{L}(H^s(\Sigma), H^{-s}(\Sigma))$ for some $s > 0$ is positive, symmetric and injective. For $z \in \mathbb{C}$ with $\mathcal{I}m\ z > 0$, one easily checks that the original problem is equivalent to a transmission problem with transmission conditions

$$B_z\, u_1 = B_z\, u_2, \quad B_{\overline{z}}\, u_1 = B_{\overline{z}} u_2,$$

where we have used an obvious notation. The iterative method we propose consists in solving at each step the Helmholtz equation in $\Omega_1$ and $\Omega_2$ with boundary conditions on $\Sigma$ ($r \in ]0, 1]$ denoting a relaxation parameter):

$$\begin{cases} B_z\, u_1^n = r\, B_z\, u_2^{n-1} + (1-r)\, B_z\, u_1^{n-1}, \\ B_{\overline{z}}\, u_2^n = r\, B_{\overline{z}}\, u_1^{n-1} + (1-r)\, B_{\overline{z}}\, u_2^{n-1}, \end{cases} \tag{1}$$

This algorithm is always convergent but converges exponentially if and only if:

$$T = \Lambda\Lambda^* \text{ where } \Lambda \text{ is an isomorphism from } L^2(\Sigma) \text{ into } H^{-1/2}(\Sigma). \tag{2}$$

Classical choices for $T$, based on local operators as in [1] or [3], are incompatible with (2) : nonlocal operators are required. To build explicitly a good operator $\Lambda$ we propose to use the operator $-\mathrm{div}_\Sigma\big(\mathbf{K}\,\nabla_\Sigma\big)$ where $\mathbf{K}$ is an integral operator with kernel $K(x, y)$ that should be a pseudo-differential operator of order -3/2. For this, $K$ must have the correct singularity when $x - y \to 0$ as suggested by Riesz potentials [2]

$$K(x, y) \sim |x - y|^\delta, \quad \delta = 1/2 \text{ in dimension 2}, \quad \delta = -1/2 \text{ in dimension 3}.$$

To avoid fully nonlocal operators leading to full matrices after space discretization, the idea is to localize $K$ around the diagonal $x = y$ using a smooth cut-off function $\chi(\rho) : \mathbb{R}^+ \to [0, 1]$ with support in $[0, 1]$ and is identically 1 for $\rho$ small enough. Finally we propose, $L > 0, \beta > 0$ and $\alpha \in \mathbb{C}$ being given

$$\Lambda = \alpha - \beta\, \mathrm{div}_\Sigma\big(\mathbf{K}_L\, \nabla_\Sigma\big), \quad \mathbf{K}_L\varphi(x) = \int_\Sigma \chi\Big(\frac{|x-y|}{L}\Big)\, |x - y|^\delta\, \varphi(y)\, dy \tag{3}$$

This operator fulfills (2) as soon as $\mathcal{I}m\ \alpha \neq 0$. Finally, we investigate, numerically and analytically, how to tune the parameters $(z, \alpha, \beta, \chi, L)$ to optimize the rate of convergence.

## References

[1] Després B. Domain decomposition method and the Helmholtz problem (Part II) In Second international conference on mathematical and numerical aspect of wave propagation phenomena, SIAM 1993, p. 197-206

[2] Stein E. Singular Integrals and Differentiability Properties of functions, Princeton Math. Series, Volume 30, Princeton University Press, 1970

[3] Gander, M., Magoulès, F., and Nataf, F. Optimized Schwarz Methods without Overlap for the Helmholtz Equation SIAM Journal on Scientific Computing, 2002, Volume 24:1, Pages 38-60

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# On Costabel's version of the trace theorem

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**Keywords:** Trace Theorem

Martin Costabel showed 1988 in [1,Lemma 3.6] that the trace operator $\gamma_0 : H^s(\mathbb{R}^n) \to H^{s-\frac{1}{2}}(\Gamma)$ for $\frac{1}{2} < s < \frac{3}{2}$ and $\Gamma$ the boundary of a Lipschitz domain $\Omega \subset \mathbb{R}^n$ is continuous. Z. Ding presented in [2] the same result for $\gamma_0 : H^s(\Omega) \to H^{s-\frac{1}{2}}(\Gamma)$ (see also [4]).
In this lecture it is shown that for a $C^{k-1,1}$–domain the trace operator $\gamma_0 : H^s(\mathbb{R}^n) \to H^{s-\frac{1}{2}}(\Gamma)$ is continuous for $\frac{1}{2} < s < k + \frac{1}{2}$ (see also [3,Theorem 4.3]). .

## References


[1] M. Costabel, *Boundary integral operators on Lipschitz domains: Elementary results.* SIAM J.Math.Anal. **19** (1988) 613–626.

[2] Z. Ding, *A proof of the trace theorem of Sobolev spaces on Lipschitz domains.* Proc.American Math.Soc. **124** (1996) 591–600.

[3] H. Harbrecht, W.L. Wendland, N. Zorii *Riesz minimal energy problems on $C^{k-1,1}$–manifolds.* Math.Nachrichten, to appear.

[4] S.E. Mikhailov *Traces, extensions and co–normal derivatives for elliptic systems on Lipschitz domains.* J.Math.Anal.Appl. **378** (2011) 324–342.


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# Ultraweak variational formulation and the Discontinuous Petrov Galerkin method

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**Keywords:** DPG method, ultraweak formulation

The Discontinuous Petrov Method (DPG) [1] combines a minimum residual approach with the so-called Ultraweak Variational Formulation (UVF) (We "stole" the name from Després, the methodology is different although close in spirit). I will use the Stokes problem to illustrate the main ideas, and discuss the well-posedness of the ultraweak formulation for a large class of problems described with systems of first order Partial Differential Equations (PDEs) for which the corresponding operator, with homogeneous boundary conditions, is bounded below in the $L^2$ sense with a stability constant $\gamma$. The corresponding UVF is then well posed as well with a mesh independent inf-sup constant of order $\gamma$. The DPG method inherits automatically the stability from the continuous level converging with optimal rates.
We will conclude with a number of numerical examples for the Stokes problem. For details, see [2].

### References

[1] L. Demkowicz and J. Gopalakrishnan, *A Class of Discontinuous Petrov-Galerkin Methods. Part II: Optimal Test Functions*, Numer. Meth. Part. D. E, 27,70-105,2011.

[2] N. Roberts, Tan Bui-Thanh and L. Demkowicz, *The DPG Method for the Stokes Problem*, ICES Report 2012/22 (in review in Comput. Math. Appl.).

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# Stability and Instability of Cycled Inverse Scattering Problems

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Instability is one of the key features of many inverse problems in geophysics. Instability is a property which can arise from the underlying dynamics systems, in particular when they are chaotic or effectively have branching points. Instabilities of data assimilation algorithms or cycled reconstruction algorithms will also come from compact or ill-posed observation operators, as they arise when inverse scattering is employed for reconstructions.

Usually, cycled reconstruction or data assimilation algorithms regularize the inversion for every time-step or cycle of the assimilation method, such that each individual reconstruction is stable. But over time instabilities are likely to occur in basically all practically applicable realizations of such systems. We will describe a mathematical framework which is suitable to study and anlyse such instabilities. In particular, we work out the behaviour of cycled data assimilation systems such as 3dVar or 4dVar explicitly for simple linear systems. Also, we show how assimilation systems can be stabilized, a general theory for self-adjoint linear dynamical systems and its interaction with inverse scattering or related inversion approaches is presented. Numerical examples are provided.

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# Integral Equations

# Singularities of solutions to mixed interface crack problems

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**Keywords:** Mixed transmission problems, Interface crack, Singularities of solution

We investigate asymptotic properties of solutions to mixed boundary value problems of thermo-piezoelectricity (thermo-electro-elasticity) for piece wise homogeneous anisotropic elastic solid structures with interior and interface cracks. Using the potential method and theory of pseudodifferential equations on manifolds with boundary we prove the existence and uniqueness of solutions. The singularities and asymptotic behaviour of the mechanical, thermal and electric fields are analyzed near the crack edges and near the curves, where different types of boundary conditions collide. In particular, for some important classes of anisotropic media we derive explicit expressions for the corresponding stress singularity exponents and demonstrate their dependence on the material parameters. The questions related to the so called oscillating singularities are treated in detail as well.
The contribution extends the results obtained in the reference [1] to more complex problems.

## References

[1] T.Buchukuri, O.Chkadua, D.Natroshvili, *Mixed boundary value problems of thermopiezoelectricity for solids with interior cracks*, Integral Equations and Operator Theory, **64**, 4 (2009), 495-537.

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# A meccano set for two dimensional waves


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**Sijiang L LU**, University of Delaware



**Keywords:** Calderón Calculus, Helmholtz equation, Nyström methods, Petrov-Galerkin methods


In this talk we will present a complete set of fully discretized integral operators for the Helmholtz equation in two dimensions, working on any fine set of smooth closed boundaries. The goal of this collection is the development of do-it-yourself tools for simultaneous use of all boundary integral operators, with automatic handling of complicated geometries, while based of very simple-minded discretization techniques that lead to extremely easy coding (with a level of difficulty not much higher than one dimensional finite differences).
The discretization is carried out by carefully choosing source and observation points, leading to a type of Nyström approximation of all operators. The choices depend on several parameters: staggering of the grids, symmetrization-and-stabilization coefficients, and quadrature weights.
We will comment on how the existing asymptotic results that justified the tilted Calderón Calculus of order two in [1] can be combined to create a fully discrete set of integral equations [2] that are of order three in all quantities, meaning that potentials are recovered with order three, but also boundary quantities are obtained with uniform order three approximations.
We will finally show how developing the same calculus for the Laplace resolvent equation leads to a fully discrete method for the transient wave equation, by combining the Nyström matrices with convolution quadrature.

**References**


[1] V. Domínguez, S.L. Lu, and F.J.Sayas, *A fully discrete Calderón Calculus for two dimensional time harmonic waves*, Int. J. Numer. Anal. Model. (to appear)

[2] V. Domínguez, S.L. Lu, and F.J.Sayas, *A Nyström flavored order three Calderón Calculus for two dimensional waves, time-harmonic and transient*, in preparation.



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# Adaptive BEM


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**Keywords:** boundary element method, adaptivity


We consider the $h$-version of the boundary element method (BEM) in 2D and 3D on shape regular meshes. We show convergence and prove quasi-optimality of an adaptive BEM (ABEM), taking Symm's integral equation as our model problem for a first kind integral equation. Optimality here means that the algorithm realizes the optimal rate achievable for solutions for an approximation class that is characterized by the best possible decay rate achievable for the error indicator under the mesh refinements allowed (here: newest vertex bisection). The error indicators that drive the adaptive algorithm are of residual type and hark back to [4, 3]. For the FEM on shape regular meshes, similar convergence and optimality results are available, [5, 2]. The BEM setting is more involved and requires different mathematical tools since the operators and pertinent norms are non-local. This mandates in particular the use of non-standard inverse estimates for integral operators, which we present in this talk. We will also discuss extensions of the algorithm to account for data approximations and applications to FEM-BEM coupling.

## References


[1] M. FEISCHL, M. KARKULIK, J.M. MELENK, D. PRAETORIUS, *Quasi-optimal convergence rate for an adaptive boundary element method* in press in *SIAM J. Numer. Anal.*

[2] J.M. CASCON, C. KREUZER, R. H. NOCHETTO K. SIEBERT. QUASI-OPTIMAL CONVERGENCE RATE FOR AN ADAPTIVE FINITE ELEMENT METHOD. *SIAM J. Numer. Anal.*, 46(5):2524–2550, 2008.

[3] Carsten Carstensen, Matthias Maischak, and Ernst P. Stephan. A posteriori error estimate and $h$-adaptive algorithm on surfaces for Symm's integral equation. *Numer. Math.*, 90(2):197–213, 2001.

[4] C. CARSTENSEN AND E. P. STEPHAN. *A posteriori error estimates for boundary element methods. Math. Comp.*, 64(210):483–500, 1995.

[5] R.P. STEVENSON. *Optimality of a standard adaptive finite element method. Found. Comput. Math.*, 7:245–269, 2007.



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# Boundary Integral Formulation of the Transmission Eigenvalue Problem for Maxwell's Equations

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**Keywords:** Integral equations, Maxwell's equation, inhomogeneous media, transmission eigenvalues

The interior transmission problem arises in inverse scattering theory for inhomogeneous media. It is a boundary value problem for a coupled set of equations defined on the support of the scattering object and appears in the analysis and solution methods for the inverse medium problem. Of particular interest is the eigenvalue problem associated with this boundary value problem, referred to as the transmission eigenvalue problem and, more specifically, the corresponding eigenvalues which are called transmission eigenvalues. The transmission eigenvalue problem is a nonlinear and non-selfadjoint eigenvalue problem that is not covered by the standard theory of eigenvalue problems for elliptic equations [2].

We consider the transmission eigenvalue problem corresponding to the electromagnetic scattering by an inhomogeneous medium assuming that the magnetic permeability in the medium is constant and the same as in the background. More specifically the transmission eigenvalue problem is formulated as finding two vector valued functions $\mathbf{E}$ and $\mathbf{E}_0$ that satisfy:

$$\begin{aligned}
\mathrm{curl}\,\mathrm{curl}\,\mathbf{E} - k^2\, N\,\mathbf{E} = 0 \qquad & \text{in} \qquad D \\
\mathrm{curl}\,\mathrm{curl}\,\mathbf{E}_0 - k^2\,\mathbf{E}_0 = 0 \qquad & \text{in} \qquad D \\
\mathbf{E}\times\nu = \mathbf{E}_0\times\nu \qquad & \text{on} \qquad \partial D \\
\mathrm{curl}\,\mathbf{E}\times\nu = \mathrm{curl}\,\mathbf{E}_0\times\nu \qquad & \text{on} \qquad \partial D
\end{aligned}$$

where $D \subset \mathbb{R}^3$ is a bounded connected region with smooth boundary $\partial D$ representing the support of the scattering media, and $N$ is the (possible matrix valued) refractive index defined in $D$. The values of $k \in \mathbb{C}$ for which this problem has nontrivial solutions are called *transmission eigenvalues*. It is well known that if the contrast $N - I$ is strictly positive or negative in $D$ the set of (possible complex) transmission eigenvalues is discrete and there exists an infinite set of of real positive transmission eigenvalues accumulating at $+\infty$ [1]. In this presentation we consider the case when contrast $N - I$ keep the same sign only in a neighborhood of the boundary $\partial D$ and can change sign inside the domain $D$. Under this relaxed assumption on the contrast in the scattering medium, we establish the Fredholm property for the transmission eigenvalue problem based on a boundary integral formulation (see [3] for the scalar case). Then following [4] we show that there is a wave number $k \in \mathbb{C}$ that is not a transmission eigenvalue, which on its turn implies that the set of transmission eigenvalue is at most discrete. Our integral equation approach makes use of the standard potentials for Maxwell's equation but in weaker trace space solution, since the solution of the above problem is $\mathbf{E} \in L^2(D)$, $\mathbf{E}_0 \in L^2(D)$ with $\mathbf{E} - \mathbf{E}_0 \in H(\mathrm{curl}\,, D)$ and $\mathrm{curl}\,(\mathbf{E} - \mathbf{E}_0) \in H(\mathrm{curl}\,, D)$. The main difficulty is to establish the correct mapping properties of the involved boundary integral operators. This is a joint work with Houssem Haddar.

## References

[1] F. CAKONI, D. GINTIDES, AND H. HADDAR, *The existence of an infinite discrete set of transmission eigenvalues* SIAM J. Math. Anal., 42, 237–255, 2010.

[2] F. CAKONI AND H. HADDAR, *Transmission Eigenvalues in Inverse Scattering Theory* Inverse Problems and Applications, Inside Out 60, MSRI Publications 2013.

[3] A. COSSONNIERE AND H. HADDAR, *Surface integral formulation of the interior transmission problem* J. Int. Eqn Appl., 43, 1698–1715, 2011.

[4] J. SYLVESTER, *Discreteness of transmission eigenvalues via upper triangular compact operators* SIAM J. Math. Anal. 44, 341-354, 2012.

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# Sobolev Spaces and Scattering by Fractal Screens

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**Keywords:** Helmholtz equation, boundary integral equation, Sobolev spaces

A significant theme in Martin Costabel's research is the correct formulation and function space setting for boundary value problems and boundary integral equations on non-smooth domains, and the implications of the choice of formulation for mathematical modelling and numerical simulation (e.g., [4, 5]). An overlapping theme is boundary integral equation methods on non-smooth domains and their numerical solution (e.g., [6]), including work on analysis and numerical methods for planar screen problems [7, 8]. In this talk we take up these themes with new twists.

The talk has two concerns. The first is the formulation and well-posedness of boundary integral equations for problems of acoustic scattering by planar screens, solving the Helmholtz equation $\Delta u + k^2 u = 0$. This is an old, well-studied problem: our new angle is to treat the case where the screen occupies a rather general bounded subset of the plane (previous work assumes that the screen is a Lipschitz subset of $\mathbb{R}^2$ or smoother, e.g., [12, 9, 7]). We have in mind the case that the screen $\Gamma$ or its boundary is fractal, this motivated by the widespread use of fractal planar antennae in an electromagnetic context [11].

The second concern, motivated by the analysis of these problems, is classical Sobolev spaces, and their duals, on general open and closed subsets of $\mathbb{R}^n$, precisely the Sobolev spaces $H^s(\Omega)$, $\tilde{H}^s(\Omega)$, and $H^s_F$, where $\Omega$ and $F$ are respectively open and closed subsets of $\mathbb{R}^n$. Here our notations are those of [10], so that $\tilde{H}^s(\Omega)$ is the closure of $C_0^\infty(\Omega)$ in $H^s(\mathbb{R}^n)$ and $H^s_F \subset H^s(\mathbb{R}^n)$ is the space of those elements of $H^s(\mathbb{R}^n)$ with support in $F$. For $s \geq 0$ we also use the notation $\mathring{H}^s(\Omega) := \{u \in H^s(\mathbb{R}^n) : u = 0 \text{ a.e. in } \mathbb{R}^n \setminus \Omega\}$.

Regarding Sobolev spaces, we show through examples that the simplifications that apply when $\Omega$ is Lipschitz, e.g. that $\tilde{H}^s(\Omega) = \mathring{H}^s(\Omega)$ for $s \geq 0$, and that $\tilde{H}^s(\Omega) = H^s_{\overline{\Omega}}$, fail in the general case. Our positive results include that $(\tilde{H}^s(\Omega))^* \cong (H^{-s}_{\mathbb{R}^n\setminus\Omega})^\perp \cong H^{-s}(\Omega)$ for an arbitrary open set $\Omega$ and that $(H^s_F)^* \cong \tilde{H}^{-s}(\mathbb{R}^n \setminus F)^\perp$ for an arbitrary closed set $F$.

Regarding boundary integral equations for the acoustic screen problem, we show that the single-layer operator $S$ and the hypersingular operator $T$ are both coercive (and so isomorphisms) on the relevant energy spaces, this extending the known results when the screen is a Lipschitz open set to cases where the screen is any bounded open or closed set. For example, we show that $T$ is coercive as an operator from $\tilde{H}^{1/2}(\Omega)$ to its dual, as an operator from $\mathring{H}^{1/2}(\Omega)$ to its dual, and as an operator from $H^{1/2}_F$ to its dual, for every bounded open set $\Omega$ and bounded closed set $F$. This leads to an interesting dilemma when solving the integral equation $T[u] = \partial u^i/\partial n$ which models the sound hard acoustic scattering problem when the screen occupies a bounded open set $\Omega$ and $u^i$ is the incident field: should we seek the solution $[u]$ (the jump in the total acoustic field across the screen) as the unique solution in $\tilde{H}^{1/2}(\Omega)$, the unique solution in $\mathring{H}^{1/2}(\Omega)$, or the unique solution in $H^{1/2}_{\overline{\Omega}}$? We show that there are screens $\Omega$ and incident fields $u^i$ such that these three solutions are all distinct: indeed the solutions in $\tilde{H}^{1/2}(\Omega)$ and $H^{1/2}_{\overline{\Omega}}$ can be distinct even where $\partial\Omega = \overline{\Omega} \setminus \Omega$ has measure zero. To illustrate how to select the physically correct solution, we show which solution is the correct limit of a sequence of scattering solutions $u_n$ corresponding to a sequence of regular screens $\Omega_n$ approaching a fractal $\Omega$. Finally, we study the wavenumber dependence of the conditions numbers of the operators $T$ and $S$, showing that (whichever function space setting is used and whatever the geometry of the screen) $\mathrm{cond}(S) = O(k^{1/2})$ and $\mathrm{cond}(T) = O(k^{2/3})$ as $k \to \infty$, this extending earlier results in [9, 1, 2].

For more detail see [3, 2] and other work in preparation.

## References

[1] T. Betcke, S. N. Chandler-Wilde, I. G. Graham, S. Langdon, and M. Lindner, *Condition number estimates for combined potential boundary integral operators in acoustics and their boundary element discretisation*, Numer. Methods PDEs, **27**, 31–69, 2011.

[2] S. N. Chandler-Wilde, I. G. Graham, S. Langdon, and E. A. Spence, *Numerical-asymptotic boundary integral methods in high-frequency acoustic scattering*, Acta Numer., **21**, 89–305, 2012.

[3] S.N. CHANDLER-WILDE, D.P. HEWETT, *Scattering by arbitrary planar screens.* In: Computational Electromagnetism and Acoustics (R. Hiptmair, R. H. W. Hoppe, P. Joly, U. Langer, Eds.) Oberwolfach Reports Vol. 10, 2013.

[4] M. COSTABEL, *A coercive bilinear form for Maxwells equations*, J. Math. Anal. Appl. **157**, 527–541, 1991.

[5] M. COSTABEL, M. DAUGE, *Weighted regularization of Maxwell equations in polyhedral domains: a rehabilitation of nodal finite elements*, Numer. Math. **93**, 239-277, 2002.

[6] M. COSTABEL, *Boundary integral operators on Lipschitz domains: elementary results*, SIAM J. Math. Anal. **19**, 613-626, 1988.

[7] M. COSTABEL, F. PENZEL, R. SCHNEIDER, *Error analysis of a boundary element collocation method for a screen problem in* $\mathbb{R}^3$, Math. Comp. **58**, 575–586, 1992.

[8] M. COSTABEL, *Time-dependent problems with the boundary integral equation method*, Chapter 25 of Encyclopedia of Computational Mechanics. (E. Stein, R. de Borst, T.J.R Hughes, Eds.) John Wiley, 703–721, 2004.

[9] T. HA-DUONG, *On the boundary integral equations for the crack opening displacement of flat cracks*, Integral Equations and Operator Theory **15**, 427–453, 1992.

[10] W. MCLEAN, *Strongly Elliptic Systems and Boundary Integral Equations*, CUP, 2000.

[11] G. SRIVATSUN, S. S. RANI, G. S. KRISHNAN, *A Self-Similar Fractal Cantor Antenna for MICS Band Wireless Applications*, Wireless Engineering and Technology **2**, 107–111, 2011.

[12] E. P. STEPHAN, *Boundary integral equations for screen problems in* $\mathbb{R}^3$, Integral Equations and Operator Theory **10**, 236–257, 1987.

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# DPG BEM with optimal test functions

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**Keywords:** Discontinuous Petrov-Galerkin method with optimal test functions, boundary element method, hypersingular operators

We present an ultra-weak formulation of a hypersingular integral equation on closed polygons and prove its well-posedness and equivalence with the standard variational formulation. Based on this ultra-weak formulation we present a discontinuous Petrov-Galerkin method with optimal test functions and prove its optimal convergence in $L^2$. We report on numerical results for an open curve with uniform and adaptively refined meshes.

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# Boundary Integral Equations on Complex Screens

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**Keywords:** Screen problems, trace spaces, boundary integral equations

A complex screen is an arrangement of panels that may not be even locally orientable because of junction points (2D) or edges (3D), see Figure 1. Whereas the situation of simple screens that are locally orientable Lipschitz manifolds with boundary is well understood, the presence of junctions compounds difficulties encountered in the definition of appropriate trace spaces and boundary integral operators associated with second-order elliptic PDEs outside the screen.

Our approach to overcome these difficulties is guided by the intuition that a screen is the limiting case of a massive object, heavily relies on the understanding of trace spaces as quotient spaces of functions defined on the complement of the screen, and employs Green's formula to define duality pairings in trace spaces.

Using these ideas and tools, we generalize the notions of trace spaces with boundary conditions and jump traces to complex screens. Taking the cue from M. Costabel's seminal work about boundary integral operators on Lipschitz domains, we introduce layer potentials and boundary integral operators for scalar second-order elliptic PDEs and derive their properties like jump relations. Extensions to electromagnetic field equations will be discussed briefly.

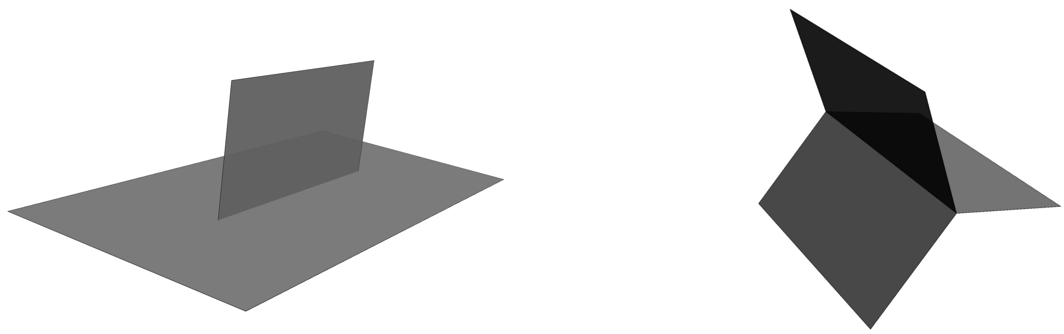

Figure 1: Two examples of complex screen geometries

## References

[1] X. Claeys and R. Hiptmair, *Integral equations on multi-screens*, Tech. Rep. 2013-03, Seminar for Applied Mathematics, ETH Zürich, Switzerland, 2013. Submitted to Integral Equations and Operator Theory.

**Ralf Hiptmair**, Seminar for Applied Mathematics, ETH Zürich, Switzerland
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# Spatial decay of time-dependent three-dimensional Navier-Stokes flows with nonzero velocity at infinity

Paul Deuring

We consider instationary incompressible Navier-Stokes flows in an exterior domain in 3D, under the assumption that the velocity at infinity is nonzero.

In view of estimating the spatial decay, uniformly in time, of the velocity part of a strong solution, we derive a representation formula for the velocity. To this end, we solve a boundary integral equation whose right-hand side not only depends on the data of the problem, but also on the solution. This situation makes it difficult to estimate this right-hand side in suitable norms, and thus solve the integral equation in question.

# Existence and uniqueness of Maxwell harmonic waves for an impedance half-space

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**Keywords:** Maxwell equations, Surface waves, Impedance Condition

We show existence and uniqueness of outgoing solutions for the Maxwell problem with an impedance boundary condition in a half-space. Due to the presence of surface waves for certain polarizations, the established Silver-Müller radiation conditions differ to the classical ones when $|x_3| < r^\gamma$ with $\gamma \in (\frac{1}{4}, \frac{1}{2})$. A half-space Green's function framework is used to carry out our results, that is obtained as a limit of the Green's function presented in [1] for the domain composed by three isotropic layers in $\mathbb{R}^3$. This extends the results obtained by Durán *et al.* for the Helmholtz equation [2]. We also discuss the numerical implications of these characterizations.

## References

[1] C. Jerez-Hanckes and J.-C. Nédélec, *Asymptotics for Helmholtz and Maxwell solutions in 3-D open waveguides.*, Commun. Comput. Phys., 11:629-646, 2012.

[2] M. Durán, I. Muga and J.-C. Nédélec, *The Helmholtz equation in a locally perturbed half-space with non-absorbing boundary*, Archive for Rational Mechanics and Analysis, 191(1):143-172, 2009.

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# A fast multipole formulation for elastic wave propagation in semi-infinite media

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**Keywords:** Fast multipole, elastodynamics, semi-infinite media, generalized Gaussian quadrature

In this communication, a version of the frequency-domain elastodynamic Fast Multipole-Boundary Element Method (FM-BEM) for semi-infinite media, based on the half-space Green's tensor (and hence avoiding any discretization of the planar unbounded part of the traction-free surface), is presented [3]. The half-space Green's tensor is known since a long time (see e.g. [5]) and is often used (in non-multipole form until now) for computing elastic wave propagation in the context of soil-structure interaction, with applications to seismology or civil engineering.
However, unlike the full-space Green's tensor [2], the elastodynamic half-space Green's tensor cannot be expressed using derivatives of the Helmholtz fundamental solution. As a result, multipole expansions of that tensor cannot be obtained directly from known expansions, and are instead derived here by means of a partial Fourier transform with respect to the spatial coordinates parallel to the free surface. The obtained formulation critically requires an efficient quadrature for the Fourier integral, whose integrand is both singular and oscillatory. Under these conditions, classical Gaussian quadratures would perform poorly, fail or require a large number of points. Instead, a version custom-tailored for the present needs of a methodology proposed by Rokhlin and coauthors [1], which generates generalized Gaussian quadrature rules for specific types of integrals, has been implemented.
The accuracy and efficiency of the proposed formulation is demonstrated through numerical experiments on single-layer elastodynamic potentials involving up to about $N = 6\,10^5$ degrees of freedom. In particular, a complexity significantly lower than that of the non-multipole version is shown to be achieved. In addition, numerical results for the complete FM-BEM formulation are given on an example involving the scattering of given incident waves by cavities buried inside a semi-infinite medium. Finally, the analytical removal in the Fourier integral of both the pole associated with the Rayleigh velocity and the pseudopole associated with the compressional wave velocity is effected in a manner that preserves the separated form of the integral (by modifying an approach proposed in [4]), and its effect on the proiposed FM-BEM formulation discussed.

### References

[1] J. Bremer, Z. Gimbutas, V. Rokhlin, *A nonlinear optimization procedure for generalized Gaussian quadrature*, SIAM J. Sci. Comput. **32**:1761–1788 (2010).

[2] S. Chaillat, M. Bonnet and J.F. Semblat, *A multi-level fast multipole BEM for 3-D elastodynamics in the frequency domain*, Comput. Meth. Appl. Mech. Eng. **197**:4233-4249 (2008).

[3] S. Chaillat and M. Bonnet, *A new Fast Multipole Formulation for the Elastodynamic Half-space Green's tensor*, under review.

[4] M. Durán, E. Godoy and J.-C. Nédélec *Theoretical aspects and numerical computation of the time-harmonic Green's function for an isotropic elastic half-plane with an impedance boundary condition*, ESAIM: Math. Model. Numer. Anal. **44**:671–692, 2010

[5] L. Pan, F. Rizzo and P.A. Martin, *Some efficient boundary integral strategies for time-harmonic wave problems in an elastic halfspace*, Comput. Meth. Appl. Mech. Eng. **164**:207–221 (1998).

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# A Time Dependent Boundary-field Equation Approach to Fluid-structure Interaction

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**Keywords:** Retarded potential, Laplace transform, Variational formulation, Sobolev space.
The problem of determining the manner in which an incoming acoustic wave is scattered by an elastic body immersed in a fluid is one of central importance in detecting and identifying submerged objects. The problem is generally referred to as a fluid-structure interaction and is mathematically formulated as a time-dependent transmission problem. In this paper, we consider a typical fluid-structure interaction problem by using a coupling procedure which reduces the problem to a nonlocal initial-boundary problem in the elastic body with integral equations on the interface between the domains occupied by the elastic body and the fluid. We analyze this nonlocal problem by the Lubich approach via the Laplace transform with an essential feature of which is that it works directly on data in the time domain instead in the transformed domain . Our results may be served as a mathematical foundation for treating time dependent fluid-structure interaction problems by convolution quadrature coupling of FEM and BEM. This talk is based on a joint work with Francisco-Javier Sayas and Richard J. Weinacht.
Communication with its references [1], [2], [3].

## References

[1] M. COSTABEL, *Time-dependent problems with the boundary integral equation method*, Chapter 25, 703–721, Volume 1 Fundamentals. In Encyclopedia of Computational Mechanics, John Wiley & Sons, Ltd. 2004.

[2] G.C. HSIAO, F.-J. SAYAS, AND R.J. WEINACHT, *A Time Dependent Fluid-Structure Interaction*, to appear.

[3] A. R. LALIENA AND F.-J. SAYAS, *Theoretical aspects of the application of convolution quadrature to scattering of acoustic waves*, Numer. Math., 112: 637–678, 2009.

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# Tensor methods for fast multidimensional integration in the Hartree-Fock calculations

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**Keywords:** 3D convolution, Hartree-Fock equation, tensor numerical methods, two-electron integrals

*Dedicated to Prof. Martin Costabel on occasion of his 65th birthday*

Tensor numerical methods provide the efficient low-parametric separable representation of multivariate data, that allows the numerical approximation of d-dimensional PDEs with linear complexity scaling in the dimension. The recent quantized tensor approximation (QTT) method [2] is proven to provide the logarithmic complexity scaling in the grid-size applied to a class of multi-dimensional functions and operators.
In this talk we show how the grid-based tensor approximation applies to the hard problems arising in electronic structure calculations, such as the multi-dimensional convolution transform [1] with the Newton kernel, two-electron integrals and the Hartree-Fock calculus for large molecular systems [3, 4, 5, 6]. The proper resolution of many local singularities in the electron orbitals enforces the discretization on large 3D grids. Our approach provides the accurate approximation of all arising functions and operators on $N \times N \times N$ tensor grids with $O(N)$ complexity, while the most computationally expensive operations can be implemented with the logarithmic cost $O(\log N)$. In the typical cases the univariate grid-size varies up to $N \approx 2^{16}$.
Numerical tests illustrate the efficiency of our tensor methods in the grid-based computations of multiple 3D convolution integrals and in the fast solution of the Hartree-Fock equation for compact molecules.

*http://personal-homepages.mis.mpg.de/bokh*

**References**

[1] B.N. Khoromskij. *Fast and Accurate Tensor Approximation of a Multivariate Convolution with Linear Scaling in Dimension.* J. of Comp. Appl. Math., **234** (2010) 3122-3139.

[2] B.N. Khoromskij. $O(d \log N)$*-Quantics Approximation of* $N$*-*$d$ *Tensors in High-Dimensional Numerical Modeling.* J. Constr. Approx. v. 34(2), 257-289 (2011).

[3] B.N. Khoromskij, V. Khoromskaia, and H.-J. Flad. *Numerical solution of the Hartree-Fock equation in multilevel tensor-structured format.* SIAM J. Sci. Comp., **33**(1), 2011, 45-65.

[4] V. Khoromskaia, D. Andrae, and B.N. Khoromskij. *Fast and Accurate 3D Tensor Calculation of the Fock Operator in a General Basis*. Comp. Phys. Communications, 183 (2012) 2392-2404.

[5] V. Khoromskaia, B.N. Khoromskij, and R. Schneider. *Tensor-structured calculation of two-electron integrals in a general basis*. SIAM J. Sci. Comput., **35**(2), 2013, A987-A1010.

[6] V. Khoromskaia and B.N. Khoromskij. *Møller-Plesset (MP2) Energy Correction Using Tensor Factorizations of the Grid-based Two-electron Integrals*. Preprint 26/2013, MPI MiS, Leipzig 2013 (submitted).

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# Boundary element methods for impedance transmission conditions

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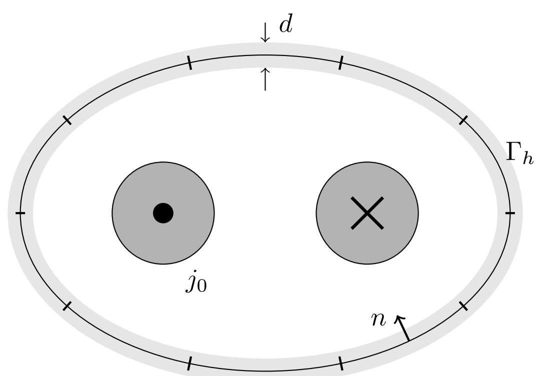
**Keywords:** Impedance transmission conditions, boundary element method

Shielding sheets are commonly used in the protection of electronic devices. With their large aspect ratios they become a serious issue for the direct application of the boundary element method (BEM) due to the occuring almost singular integrals.
Impedance transmission conditions (ITCs), posed on the sheet mid-line (or mid-surface) $\Gamma$, boundary element formulations on $\Gamma$ only. The impedance transmission conditions can be written in the general form [1, 2]

$$0 = T_{11}\,[\gamma_0 U] + T_{12}\,\{\gamma_0 U\} + T_{13}\,\{\gamma_1 U\},$$
$$0 = T_{21}\,[\gamma_1 U] + T_{22}\,\{\gamma_0 U\} + T_{23}\,\{\gamma_1 U\},$$

where $\{\cdot\}$ and $[\cdot]$ are the mean or jump, respectively, and $T_{ij}$ are (possibly differential) operators depending on frequency, conductivity, sheet thickness and sheet geometry (*e.g.* curvature). The operators $T_{ij}$ may include parameters which may take small or large values.
We will propose boundary integral equations (BIE) and boundary element methods [3] of different types of impedance transmissions conditions in two and three dimensions and analyse the well-posedness and discretisation error depending on the model parameters. The analysis deals with the case of singular perturbed BIEs, where some smoothness assumption on $\Gamma$ and an asymptotic expansion of the solution of the BIE or the BEM in the small parameter lead to the explicit estimates. Numerical experiments confirm the convergence order of the discretisation error of the proposed BEM and that discretisation error behaves equivalent to the exact solution when varying the model parameters. The results obtained for the eddy current model, for which a Poisson equation has to be solved outside $\Gamma$, can be transfered to the Helmholtz equation and to transmission conditions arising from other models.




Mesh $\Gamma_h$ of the mid-line/mid-surface $\Gamma$ for boundary element methods for impedance transmission conditions.

## References

[1] K. Schmidt and S. Tordeux, “High order transmission conditions for thin conductive sheets in magneto-quasistatics”, ESAIM: Math. Model. Numer. Anal., vol.45, no.6, pp. 1115–1140, 2011.

[2] K. Schmidt and A. Chernov, “A unified analysis of transmission conditions for thin conducting sheets in the time-harmonic eddy current model,” Preprint 2012-36, Inst. f. Mathematik, TU Berlin, 2012.

[3] K. Schmidt and R. Hiptmair, “Asymptotic boundary element methods for thin conducting sheets”, in preparation.

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# MELLIN CONVOLUTION OPERATORS IN BESSEL POTENTIAL SPACES WITH ADMISSIBLE MEROMORPHIC KERNELS

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**Keywords:** Mellin convolution, Bessel potential, Meromorphic kernel, Banach algebra, Symbol, Fredholm Property, Index.

We study the following Mellin convolution euqtions with meromorphic kernels

$$d_0\varphi(t)+d_1\int_0^\infty\frac{\varphi(\tau)d\tau}{\tau-t}+\sum_{j=1}^n d_{j+1}t^{k_j}\int_0^\infty\frac{\tau^{m_j-k_j}\varphi(\tau)d\tau}{(t-c_j\tau)^{m_j+1}}=g(t),\qquad 0<t<\infty, \tag{1}$$

where $d_0,\dots,d_{n+1},c_1,\dots,c_n\in\mathbb{C}$ are complex numbers, $\operatorname{Im}c_j\neq 0$ for $j=1,\dots,n$ and $0\leq k_j\leq m_j$ are integers, $j=1,\dots,n$. Such equations in Bessel potential spaces setting $\varphi\in\widetilde{\mathbb{H}}^s_p(\mathbb{R})$, $g\in\mathbb{H}^s_p(\mathbb{R})$ encounter rather often, e.g. when we deal with boundary value problems for elliptic equations in planar 2D domains with angular points on the boundary (scattering of acoustic and electromagnetic waves by piecewise-smooth boundary, deformation of plates with non-smooth boundary) or one dimensional boundary (cracks in elastic media, elastic plates reinforced with rigid bars, scattering of acoustic and electromagnetic waves by a piece-wise smooth curve; cf., e.g., [1, 2, 3]). While studying such problems with potential method, after localization we encounter model boundary Mellin convolution equations with meromorphic kernels (1), which we have to study in Bessel potential spaces on the half axes $\widetilde{\mathbb{H}}^s_p(\mathbb{R}^+)$ and in the setting $\widetilde{\mathbb{H}}^s_p(\mathbb{R}^+)\to\mathbb{H}^s_p(\mathbb{R}^+)$.

Lifting a Mellin convolution equation (1) from Bessel potential space setting to Lebesgue space setting, we get equation $\mathbf{A}\psi=h$, with $\psi,\ h\in\mathbb{L}_p(\mathbb{R})$, where the operator $\mathbf{A}$ belongs to the Banach algebra $\mathfrak{A}_p$ generated by Mellin convolution and Fourier convolution operators in Lebesgue space $\mathbb{L}_p$.

Our study is based upon two results. The first is new and concerns commutants of Mellin convolution and Bessel potential operators. It is shown that Bessel potentials alter essentially after commutation with Mellin convolutions depending on the poles of the kernel. For example, if $0<|\arg\gamma|<\pi$ and $-\pi<\arg(\gamma\,c)<0$, then

$$\Lambda^s_{-\gamma}\mathbf{K}^m_c=c^{-s}\mathbf{K}^m_c\Lambda^s_{-\gamma\,c},$$

where

$$\mathbf{K}\varphi(t)=\int_0^\infty\frac{\tau^m\varphi(\tau)d\tau}{(t-c\tau)^{m+1}}$$

and $\Lambda^s\mathcal{F}^{-1}(\xi-\gamma)^r\mathcal{F}$ is the Bessel potential.

The second basic ingredient is the Fredholm criteria for operators from the Banach algebra $\mathfrak{A}_p$, obtained by the author in 1970's and 1980's (see, e.g., [4]). These results are modified by adding to the algebra $\mathfrak{A}_p$ the Hankel operators, which encounter in applications. Hankel operators are actually contained in the algebra $\mathfrak{A}_p$, but we indicate their symbol precisely.

Examples of Mellin convolution operators which encounter in applications are considered.

The obtained results will be applied in forthcoming papers to the study of boundary value problems for the Helmholtz equations in composite domains with angular points on the boundary and different scattering properties of neighbouring sub-domains.

## References

[1] R. DUDUCHAVA, *Integral equations with fixed singularities*, Teubner, Leipzig, 1979.

[2] R. DUDUCHAVA, *An application of singular integral operators to some problems of elasticity*, Integral Equations Operator Theory **5** (1982) 475-489.

[3] R. DUDUCHAVA, *On general singular integral operators of the plane theory of elasticity*, Rend. Sem. Mat. Univ. Politec. Torino **42** (1984) 15-41.

[4] R. DUDUCHAVA, *On algebras generated by convolutions and discontinuous functions*, Integral Equations Operator Theory **10** (1987) 505-530.

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# Rellich estimates at the boundary in $L^p$ for elliptic systems

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**Keywords:** elliptic systems, boundary value problems

We wish to report on recent work with M. Mourgoglou [1], but also on the current development, in preprint form [2], on boundary value problems for elliptic systems on divergence forms with measurable coefficients. We will attempt to describe some relations between different sets of boundary inequalities which usually carry the name of Rellich who invented the notion for different type of problems.

## References

[1] P. AUSCHER, AND M. MOURGOGLOU, *Boundary layers, Rellich estilates and extrapolation of solvability for ellliptic systems*, arXiv:1306.4115.

[2] S. HOFMANN, C. KENIG, S. MAYBORODA, AND J. PIPHER, *The Regularity problem for second order elliptic operators with complex-valued bounded measurable coefficients*, arXiv:1301.5209.

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# Integral operators over a unit disc in $R^3$.

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**Keywords:** Integral equations

We introduce explicit and exact variational formulations for some weakly- and hyper-singular operators over an open flat circle as well as for their corresponding inverses. Contrary to the case of a closed domain, these operators no longer map fractional Sobolev spaces in a dual fashion but degenerate into different subspaces depending on their extensibility by zero. We show that an average and jump decomposition leads to precise coercivity results and characterize the mismatch occurring between associated functional spaces. Through this setting, we naturally define Calderón-type identities with their potential use as preconditioners. Moreover, we provide an interesting relation between these operators and two-dimensional Laplace Dirichlet and Neumann problems on the circle.

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# Exponential fast convergence on complicated geometries with BEM

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**Keywords:** Integral equations, exponential fast convergence

The hp-method on geometrical refined meshes for 2d- and 3D-BEM leads to exponential fast convergence on polygonal and polyhedral domains [3, 5, 6] with respect to the number of unknowns. Achieving exponential fast convergence on curved surfaces asymptotically proves to be surprisingly difficult, due to requirements on memory, solution time and the necessary numerical precision [2, 7]. Here we will make use of a general surface parametrisation, cf. [4], we will allow non-matching grids, cf. [1], to simplify the connection of separately meshed parts of the surface, and we will discuss further optimization techniques to keep the complexity and numerical constants low. Emphasize is given to balancing the computational costs for evaluation of the surface parametrisation, incorporating the constraints on the spline space due to hanging nodes, with the exponential fast convergence of the method. We will present several examples where we obtain exponential fast convergence on complicated geometries.

### References


[1] L. Demkowicz, K. Gerdes, C. Schwab, A. Bajer, and T. Walsh, *HP90: A general and flexible Fortran 90 hp-FE code*, Comput. Visual. Sci., 1 (1998), pp. 145–163.

[2] S. Erichsen and S. A. Sauter, *Efficient automatic quadrature in 3-d Galerkin BEM*, Computer Methods in Applied Mechanics and Engineering, 157 (1998), pp. 215–224.

[3] B. Guo, and I. Babuška, *The h-p version of the finite element method, part 1: The basic approximation results*, Computational Mechanics, 1 (1986), 21–41.

[4] H. Harbrecht and R. Schneider, *Wavelet galerkin schemes for boundary integral equations*, SIAM J. Sci. Comput., 27 (2006), pp. 1347–1370.

[5] N. Heuer, M. Maischak, and E. P. Stephan, *Exponential convergence of the hp-version for the boundary element method on open surfaces*, Numer. Math., 83 (1999), pp. 641–666.

[6] H. Holm, M. Maischak, and E. P. Stephan, *Exponential convergence of the $h-p$ version bem for mixed bvp's on polyhedrons*, Math. Meth. Appl. Sci., 31 (2008), pp. 2069–2093.

[7] S. A. Sauter and C. Schwab, *Quadrature for hp-Galerkin BEM in R3*, Numerische Mathematik, 78 (1997), pp. 211–258.


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